## Pre-Calculus Texas Mathematics: Unpacked Content

## What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

## What is in the document?

Descriptions of what each standard means a student will know, understand, and be able to do.
The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

## At A Glance:

## New to Grade:

- Behavior of Rational Functions description went from a general description of significant values, continuity, end behavior, and limits of all functions to behavior around all asymptotes (vertical, horizontal, and oblique).
- Discontinuities - have their own TEK - also mentions the problem of the limitations of the graphing calculator
- Left-sided and right-sided equation around asymptotes
- Values of trig functions at special angles (was glossed over before - not addressed, but assumed to be necessary)
- Parametric equations used to model all real-life situations, not just motion
- Polar coordinates and equations
- Ellipses and hyperbolas (algebra 2)
- Linear and angular velocity
- Vector addition, subtraction and scalar multiplication specified both symbolically and geometrically


## Moved from Grade:

- Parabolas and circles as conic sections


## Instructional Implications for 2015-16:

Most of the changes are providing more specific guidance as to what was meant by the original TEKS. The only major implication is the transfer of the conic sections material. Students will come to Pre-calculus with much less (none in some cases) previous knowledge of the conic sections - especially the ellipse and hyperbola, whereas they had previously studied all of the conic sections in Algebra 2 and were extending them through parametric and polar equations.

## Professional Learning Implications for 2015-16:

Education of teachers as to the changes to be expected

## Pre-Calculus Primary Focal Areas:

## The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

Pre-Calculus is the preparation for Calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Pre-Calculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.

| Mathematical Process Standards <br> The student uses mathematical processes to acquire and demonstrate mathematical understanding. <br> The student is expected to: |  |
| :--- | :--- |
| $\mathbf{1 ( A )}$ apply mathematics to problems arising <br> in everyday life, society, and the workplace; | Apply |

## Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

| 1(B) use a problem-solving model that | Use, Formulate, Determine, Justify, Evaluate |
| :--- | :--- |
| incorporates analyzing given information, |  |
| formulating a plan or strategy, determining a |  |
| solution, justifying the solution, and |  |
| evaluating the problem-solving process and |  |
| the reasonableness of the solution; |  |

## Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

1(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

Select, Solve

Mathematical Process Standards
The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
1(D) communicate mathematical ideas, $\quad$ Communicate reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

## Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
1(E) create and use representations to organize, record, and communicate mathematical ideas;

## Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

1(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

Analyze
Create, Use
yze


## Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
1(G) display, explain, and justify
mathematical ideas and arguments using precise mathematical language in written or oral communication.

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(A) use the composition of two functions to model and solve real-world problems;
2(B) demonstrate that function composition is not always commutative;
2(C) represent a given function as a composite function of two or more functions;

1. Students will represent a situation using a function within a function.
2. Students will find $f(g(x))$ and $g(f(x))$ and realize that they are not always the same.
3. Students will be able to identify when one function is within another.

Example 1: You shine a flashlight, making a circular spot of light with radius 5 cm on a wall. Suppose that, as you back away from the wall, the radius of the spot of light increases at a rate of $7 \mathrm{~cm} / \mathrm{sec}$. Write an equation for $R(t)$, the radius of the light $t$ seconds after you started backing away. Use the formula, $\mathrm{A}=\pi r^{2}$ to create $A(R(t))$, the area of the circle illuminated by the flashlight as a function of time, $t$.

Solution: $R(t)=5+7 t$
$A(R(t))=\pi(5+7 t)^{2}$

Example 2: Given the functions $f(x)=5 x$ and $g(x)=x^{2}$, find $f(g(x))$ and $g(f(x))$.

|  | Solution: $f(g(x))=5 x^{2}$ and $g(f(x))=25 x^{2}$. <br> Therefore, composition of functions is not <br> commutative. <br> Example 3: If $h(x)=f(g(x))$ and $h(x)=$ <br> $\sqrt{x^{2}+4}$, find $f(x)$ and $g(x)$. <br> Solution: If we let $g(x)=x^{2}+4$ and $f(x)=$ <br> $\sqrt{x}$, then $f(g(x))=\sqrt{x^{2}+4}=h(x)$. |
| :--- | :--- |

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:
2(D) describe symmetry of graphs of even
and odd functions;

1. The student looks at either a graph or a function and determines whether it is even, odd or neither.
2. Graphically, an even function will be symmetric across the $y$-axis.
3. Graphically, an odd function has rotational symmetry of $180^{\circ}$ about the origin.
4. Analytically, if you find $f(-x)$ and $f(-x)=$ $f(x)$, then the function is even. If you find $f(-x)$ and $f(-x)=-f(x)$, then the function is odd.

Example 1: The function graphed is even because is it symmetrical across the $y$-axis.
Example 2: The function graphed is odd
because it has $180^{\circ}$ rotational symmetry
about the origin.
Example 3: Determine algebraically
whetherf $(x)=-3 x^{2}+4$ is even, odd, or
neither.
Solution:
$f(-x)=-3(-x)^{2}+4$
$=-3\left(x^{2}\right)+4$
$=-3 x^{2}+4$

|  | This is exactly opposite of $f(x)$, so this <br> function is odd. |
| :--- | :--- |

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

1. The student looks at the graph of a function and determines if it passes the horizontal line test, meaning its inverse is also a function. This also means that the function is one-to-one.
2. The student limits the domain of a function that is not one-to-one so that it will be one-to-one.
3. The student finds the inverse of a function by switching the $x$-and $y$-coordinates, then solving for y .
4. The student graphs the inverse function by reflecting the graph across the line $y=x$.
5. The student takes coordinates of a function from a table, a graph, a list or ordered pairs or a mapping notation and finds corresponding coordinates of the inverse.
6. The student proves that two functions, $f(x)$ and $g(x)$, are inverses of each other by finding $f(g(x))$ and $g(f(x))$ and showing that they both equal $x$.

Example 1: Determine if the function shown is one-to-one. If it is, graph $^{-1}(x)$. If it is not, limit its domain so that it is.
a) The function is one-to-one because it passes both the vertical and the horizontal line tests.



## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(F) graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

2(G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $a f(x), f(x)+d, f(x-$ c), $f(b x)$ for specific values of $a, b, c$, and $d$, in mathematical and real-world problems

2(H) graph $\arcsin x$ and $\arccos x$ and describe the limitations on the domain;

1. The student will graph exponential, logarithmic, polynomial, power, trigonometric, and inverse trigonometric functions.
2. The student will graph piecewise defined functions and step functions such as the greatest integer function.
3. The student will transform exponential, logarithmic, sine, cosine, rational, polynomial and power functions with vertical and horizontal stretches and shifts.
4. The student will graph the inverse sine and cosine functions, understanding that their domains of sine and cosine are limited to

fis $f(x)=\sin x$


## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

2(J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

1. The student will determine the domain, range, $y$-intercept and asymptote and end behavior of exponential functions. They will determine whether the function is increasing or decreasing.
2. The student will determine the domain, range, x-intercept end behavior and asymptote of logarithmic functions.
3. Students will determine the maximums, minimums, zeros, increasing intervals and decreasing intervals of polynomial and power functions and describe the end behavior using infinity notation.
4. The student will determine the domain, range, asymptotes and zeros of the six trig functions.

Example 1: Find the domain, range, y-intercept, asymptote and end behavior of the function.

$$
y=8(2)^{x-1}+3
$$

Solution: The domain will be all real numbers. The range will be $\{y \mid y>3\}$ since the graph will be shifted up 3 units. The $y$-intercept will be 7. The only asymptote will be $y=3$. The end behavior will be described as $\mathrm{x} \rightarrow-\infty, \mathrm{y} \rightarrow$ 3and as $\mathrm{x} \rightarrow$ $\infty, y \rightarrow \infty$.

Example 2: Will the function $y=\left(\frac{1}{3}\right)^{x}$ be increasing or decreasing?

Solution: Since $\left|\frac{1}{3}\right|<1$, the function will represent exponential decay, and will decreasing on all real numbers

Example 3: Find the domain, range, asymptote,


## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

1. The student will determine for what values of $x$ the function is undefined and determine which values are vertical asymptotes and which are removable discontinuities (holes).
2. The student will determine whether the end behavior asymptote is horizontal or oblique and find it.
3. At all vertical asymptotes, determine whether the function is going to infinity or negative infinity.
4. Determine if the function crosses the horizontal or oblique asymptote.

Example: Analyze the graph of
$f(x)=\frac{x^{2}-3 x-10}{2 x^{2}+3 x-2}$, determining all
asymptotes and intercepts.
Solution: Factoring both the numerator and the denominator yields $f(x)=\frac{(x-5)(x+2)}{(2 x-1)(x+2)}$.
Since $x+2$ is a factor of both the numerator and the denominator, there will be a removable discontinuity at $x=-2$. The $y$-coordinate of this hole will be $\frac{-2-5}{2(-2)-1}=\frac{-7}{-5}=\frac{7}{5}$. Since this is a hole, there is not a vertical asymptote at $x=-2$. There is a vertical asymptote at $x=1 / 2$ since that value will make the denominator 0 . There is an $x$ intercept at $(5,0)$. There is a horizontal asymptote of $\mathrm{y}=1 / 2$. There is a y -intercept at (0, 5).

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(L) determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

Example: Analyze the graph of
$f(x)=\frac{x^{2}-3 x-10}{2 x^{2}+3 x-2}$, determining all asymptotes and intercepts.

Solution: Factoring both the numerator and the denominator yields
$f(x)=\frac{(x-5)(x+2)}{(2 x-1)(x+2)}$.
Since $x+2$ is a factor of both the numerator and the denominator, there will be a removable discontinuity at $x=-2$. The $y$-coordinate of this hole will be $\frac{-2-5}{2(-2)-1}=\frac{-7}{-5}=\frac{7}{5}$. Since this is a hole, there is not a vertical asymptote at $x=-2$. There is a vertical asymptote at $x=1 / 2$ since that value will make the denominator 0 . There is an $x$ intercept at $(5,0)$. There is a horizontal asymptote of $y=1 / 2$. There is a $y$-intercept at $(0,5)$.

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;

The students should be able to describe what happens at infinity and negative infinity of a function. (end behavior).
Example: Analyze the graph of

$$
f(x)=\frac{x^{2}-3 x-10}{2 x^{2}+3 x-2}, \text { determining all }
$$

|  | asymptotes and intercepts. <br> Solution: Factoring both the numerator and <br> the denominator yields <br> $f(x)=\frac{(x-5)(x+2)}{(2 x-1)(x+2)}$. <br> Since $x+2$ is a factor of both the numerator and <br> the denominator, there will be a removable <br> discontinuity at $x=-2$. The $y$-coordinate of this <br> hole will be $\frac{-2-5}{2(-2)-1}=\frac{-7}{-5}=\frac{7}{5}$. |
| :--- | :--- |
| there ince not a vertical asymptote at $x=-2$. There <br> is a vertical asymptote at $x=1 / 2$ since that value <br> will make the denominator 0. There is an $x-$ <br> intercept at $(5,0)$. There is a horizontal <br> asymptote of $y=1 / 2$. There is a $y$-intercept at <br> (0, 5). |  |

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems;

1. The student will use exponential functions to solve exponential growth and decay problems and use logarithms to solve some of these problems.
2. The student will use rational, polynomial, and power functions to model real-world situations.

## Example:

If $\$ 2000$ is invested at $5.2 \%$, compounded continuously, how long will it take to double?

## Solution:

Solve the equation:

$$
\begin{gathered}
4000=2000 e^{.052 t} \\
2=e^{.052 t} \\
\ln (2)=.052 t \\
t=\ln (2) / .052 \\
\hline
\end{gathered}
$$

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(0) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and

The student should be able to write sine or cosine equations to represent appropriate situations (like Ferris wheel problems), then use the equation to predict future or past values.

## Example:

FERRIS WHEEL (Paul Foerster)
As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. You are the last seat filled and the Ferris wheel starts immediately. Let $t$ be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 ft . above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 ft .
a) Sketch a graph.
b) What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?
c) Write an equation.
d) Predict your height above the ground when: i) $t=6$; ii) $t=13 / 3$; iii) $t=0$
e) What is the value of $t$ the second time you are 18 ft above the ground?

## Functions

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

2(P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and realworld problems.

Determine the values of sine, cosine, tangent, secant, cosecant and cotangent at multiples of 30 degrees and 45 degrees using the unit circle.


Students will use the unit circle and its relationship to the sine and cosine functions to find the position of teeth around a 2-inch diameter cogwheel using ordered pairs at specified degrees around the circle.

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(A) graph a set of parametric equations;
Graph a set of parametric equations by choosing values of $t$, then finding the $x$ - and $y$-coordinates that result.

Example 1:

$$
x=3-t
$$

$y=-t \quad 2$ where $t \in[-2,4]$
(

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(B) convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;

The student should be able to take a set of parametric equations and write them as a single equation relating $x$ and $y$ without $t$.

## Example 1:

$\mathrm{x}=4 \mathrm{t}-5 ; \mathrm{y}=3-8 \mathrm{t}$
$x=4 t-5$
$y=3-8\left(\frac{x+5}{4}\right)$
$x+5=4 t$
$t=\frac{x+5}{4}$

$$
y=3-2(x+5)
$$

$$
y=3-2 x-10
$$

$$
y=-2 x-7
$$

## Example 2:

$$
\begin{array}{ll} 
& x=5 \cos t-2 ; y=5 \sin t+3 \\
& x+2=5 \cos t ; y-3=5 \sin t \\
(x+2)^{2}=25 \cos ^{2} t ;(y-3) \quad{ }^{2}=25 \sin & { }^{2} t \\
(x+2)^{2} & \left.{ }^{2}+(y-3)\right)^{2}=25\left(\cos ^{2} t+\sin \right. \\
& \left.{ }^{2} t\right) \\
& (x+2)^{2}+(y-3){ }^{2}=25 \\
\hline
\end{array}
$$



## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(C) use parametric equations to model and solve mathematical and real-world problems;

Use parametric equations to add in the parameter of time to solve problems

Example 1:
Suppose that two ships are on the following courses:

HMS Petticoat:

$$
\begin{aligned}
& x=t \\
& y=\frac{1}{2} t+30
\end{aligned}
$$

USS Junction:

$$
\begin{aligned}
& x=80-t \\
& y=t+10
\end{aligned}
$$

Determine if these two ships are going to collide. If they are, when will they collide?

They will collide because they both will be at the same location $(40,50)$ when $t=40$.

Example 2 (Projectile Motion):

|  | $x=v_{0} t \cos \theta$ |
| :--- | :--- |
| $y=v_{0} t \sin \theta-16 t^{2}+h_{0}$ |  |
| A golfer hits a ball with an initial velocity of 133 |  |
| ft/sec and at an angle of 36 degrees from the |  |
| horizontal. Find when and where the ball will |  |
| hit the ground. |  |

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(D) graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;

The student should be able to graph coordinates of the form ( $r, \theta$ ) on a polar grid, as well as be able to identify that same location using a rectangular grid and ( $\mathrm{x}, \mathrm{y}$ ) coordinates.

- Given a set of coordinates ( $x, y$ ),

$$
r=\sqrt{x^{2}+y^{2}} \text { and } \theta=\tan -1 \frac{y}{x}
$$

- Given a set of coordinates $(r, \theta)$,
$x=r \cos \theta, y=r \sin \theta$


## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:
3(E) graph polar equations by plotting points and using technology;

Graph polar equations of the following types by hand, using polar grids:

$$
\begin{aligned}
& r=a \\
& r=a \theta \\
& r=a \sin b \theta \\
& r=a \cos b \theta
\end{aligned}
$$



## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(F) determine the conic section formed when a plane intersects a double-napped

Determine what type of conic section is formed by the intersection of a plane and a
Cone;

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(G) make connections between the locus definition of conic sections and their equations in rectangular coordinates;

The student should understand how the definition of the locus of conic sections is related to the equation of each conic section using rectangular coordinates.

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(H) use the characteristics of an ellipse to write the equation of an ellipse with center (h, k); and

The student should be able to write the equation of an ellipse with the center at (h, k).

## Example:

The roof of an arena is shaped like an ellipse whose major axis measures 624 feet and whose minor axis measures 397 feet. Write the equation of this ellipse.

## Relations and Geometric Reasoning

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

3(I) use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k).

The students should be able to write the equation of a hyperbola with the center at (h, k).

Example:
Write the equation of a hyperbola whose center is at $(4,-3)$, the vertices are at $(7,-3)$ and $(1,-3)$, and whose foci are at $(9,-3)$ and (-$1,-3)$.

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(A) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;

The student should be able to use the unit circle values to solve trigonometric functions in mathematical and real-world problems.


## Example:

Betty Lou was sitting on the front porch of her plantation when the riverboat went by. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d from the water's surface was a sinusoidal function of time. Four seconds later, the point was at its highest, 16 feet above the water's surface. The diameter of the wheel was 18 feet, and it completed a

|  | revolution every 10 seconds. <br> a) Sketch a graph and find the period, phase <br> shift, amplitude and horizontal shift. |
| :--- | :--- |
|  | b) Write the equation of the sinusoid. |
| c) How far above the surface was the point after |  |
| (i) 5 seconds, (ii) 17 seconds? |  |

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:
\(\left.$$
\begin{array}{|l|l|}\hline \text { 4(B) describe the relationship between } \\
\text { degree and radian measure on the unit circle; }\end{array}
$$ \begin{array}{l}The student should be able to describe what <br>
a degree is and what a radian is on the unit <br>
circle. <br>
Example: <br>
\bullet <br>
- Consider the circle with radius 1 . <br>
The circumference of that circle is <br>
2^{*} pi. <br>
- Every circle has 360 degrees. <br>
- So, 360 degrees=2^{*} pi. The left side is <br>
in degrees and the right side is in <br>

radian.\end{array}\right\}\)| Hence 1 radian $=360 /(2$ pi) and 1 |
| :--- |
| degree $=2$ pi/360. |

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(C) represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;

The student should be able to represent angles in radians or degrees.

|  |   <br> In each drawing, 6 is the reference angle for $\vartheta$. |
| :---: | :---: |

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(D) represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;

The student should be able to angles in both radians and degrees in standard position and then use them to solve problems. They should be able to calculate arc length and sector area.



## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(E) determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;

The student should be able to find sine, cosine and tangent ratios and use them to find sides and angles of triangles - particularly in right triangles.

## Example 1

A 5 foot ladder leans against a wall. The base of the ladder is 2.5 feet from the wall on the ground. What is the angle the ladder makes with the ground?

## Solution

$$
\begin{gathered}
\cos \frac{2.5}{5}=\theta \\
\theta=60 \circ
\end{gathered}
$$

|  | Example 2 <br> A right triangle has a 36oangle. If the <br> hypotenuse of this triangle is 10 cm, find the <br> lengths of the legs. <br> Solution <br> Since one of the acute angles is 36 degrees, <br> the other one is $90-36=54$. Using either one <br> of these angles, set up ratios to find the <br> missing legs. sin36 $=\frac{a}{10^{\prime}}$, so a $=5.88$ and <br> $\cos 36=\frac{b}{10^{\prime}}$, so $b=8.09$ |
| :--- | :--- |

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(F) use trigonometry in mathematical and real-world problems, including directional bearing;

## Example:

A plane travels 38 degrees due north at 230 mph . At the same time, another plane departs due north at 315 degrees at 400 mph. How far apart are they after 2 hours.

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

| 4(G) use the Law of Sines in mathematical <br> and real-world problems; | Use the Law of Sines <br> Example: <br> You are standing 40 meters from the base of <br> a tree that is leaning 8 degrees from the <br> vertical from you. The angle of elevation <br> from your feet to the top of the tree is 20 <br> degrees. Find the height of the tree. |
| :--- | :--- |

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(H) use the Law of Cosines in mathematical
and real-world problems;

## Example:

Scientists can use a set of footprints to calculate an organism's step angle, which is a measure of walking efficiency. The closer the step angle is to 180 degrees, the more efficiently the organism walks.


The diagram shows the set of footprints for a dinosaur. Find the step angle B.

## Example:

Three towns are located in a triangular pattern. Town A is 23 miles from Town B. Town B is 30 miles from Town C. Town $C$ is 27 miles from Town A. Assuming that the roads between the towns are perfectly straight, find the angle made by the roads at Town A.

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:
4(I) use vectors to model situations involving $\quad$ Example:
magnitude and direction;

Find the magnitude and direction of the vector whose endpoints are: $(3,18)$ and $(-2,4)$.

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

## 4(J) represent the addition of vectors and the Example:

multiplication of a vector by a scalar
geometrically and symbolically; and

Given: $z=<4,3>, y=<2,-1>$

1) Find $z+y$.
2) Find $2 z$.

## Number and Measure

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

4(K) apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

## Example:

You and your friend are going to the mall. You drive to your friend's house which is 3 miles north and 2 miles west of you. If your house is located at the origin, draw the vector that represents your trip.

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(A) evaluate finite sums and geometric series, when possible, written in sigma notation;

Summation formula for arithmetic series:

$$
\text { 回 }=\frac{\text { ? }}{\text { ? }}
$$



## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(B) represent arithmetic sequences and geometric sequences using recursive formulas;

Recursive formula for arithmetic sequence:

$$
a_{n}=a_{1}+d(n-1)
$$

Recursive formula for geometric sequence:

$$
a_{n}=a_{n-1} \cdot r
$$

## Example:

Consider a sequence where the first term is $a_{1}$ and whose common difference is $d$. Complete the following table.

| Term number <br> $(n)$ | 1 | 2 | 3 |  |
| :--- | :---: | :---: | :---: | :---: |
| Term | $a_{1}+(0) d$ | $a_{1}+(1) d$ | $a_{1}+(2) d$ |  |
|  |  |  |  |  |

Write the recursive formula for the $n^{\text {th }}$ term.

Example:

|  | The MSRP for a 2004 Jeep Grand Cherokee is <br> $\$ 30,000$. Analysts estimate that prices will <br> increase 6\% per year for the next five years. <br> Find the price to the nearest dollar for this <br> model for years 2005 through 2009. Write an <br> explicit formula for this sequence. |
| :--- | :--- |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

| 5(C) calculate the $n^{\text {th }}$ term and the $\mathrm{n}^{\text {th }}$ partial <br> sum of an arithmetic series in mathematical <br> and real-world problems; | Summation formula for arithmetic series: <br> Example: <br> $5+4 \frac{1}{2}+4+\ldots .$. to twenty terms <br> Example: <br> Suppose that you are responsible for predicting <br> the cost of constructing a new multi-story office <br> building. The cost per square meter of floor <br> space for constructing the higher stories <br> increases because it is more difficult to build <br> them. The cost per square meter forms an <br> arithmetic sequence. The first story costs $\$ 400$ <br> per square meter and the fifth story costs $\$ 500$ <br> per square meter. What is the total cost per <br> square meter for constructing a 48-story <br> building? |
| :--- | :--- | :--- |


| Algebraic Reasoning <br> The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to: |  |
| :---: | :---: |
| 5(D) represent arithmetic series and geometric series using sigma notation; | Example: $20+45+70+\ldots .+n$ <br> is the same as: $S_{n}=20+25(n-1)$ <br> is the same as: $\sum_{\text {an }}^{\text {a }} \text { ( }$ <br> Example: $6+3+\frac{3}{2}+\ldots \ldots n$ <br> is the same as: $S_{n}=6\left(\frac{1}{2}\right)^{n-1}$ <br> is the same as: $\sum_{\square=0}^{a} \text { (en }$ |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(E) calculate the $\mathrm{n}^{\text {th }}$ term of a geometric series, the $n^{\text {th }}$ partial sum of a geometric series, and sum of an infinite geometric series when it exists;
(Partial) Summation formula for a geometric series:

$$
S_{n}=\frac{a_{1}\left(1-r^{n}\right)}{1-r}
$$

Formula for an infinite sum of a geometric series:
$S=\frac{a_{1}}{1-r}$ when $|r|<1$.


## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:
5(F) apply the Binomial Theorem for the
expansion of $(a+b)^{n}$ in powers of $a$ and $b$ for
a positive integer $n$, where $a$ and $b$ are any
numbers;

The Binomial Theorem for Expansion is a relatively quick way to multiply binomials to a power. Following is a formula for doing that.

$$
\begin{aligned}
& \qquad(a+b)^{n}=\sum_{k=0}^{n}\binom{n}{k} a^{n-k} b^{k} \\
& \text { Where }\binom{n}{k}={ }_{n} C_{k}=\frac{n!}{(n-k)!k!}
\end{aligned}
$$

This is also where Pascal's Triangle comes in. Use the appropriate row of Pascal's Triangle as the coefficients of the expansion. For example: if you want to expand $(a+b)^{4}$, you would use the 4th row of Pascal's Triangle:

|  | as the coefficient's for the expansion, and the a <br> and $b$ would have the appropriate exponents like so: |
| :--- | :--- |
| $1(\mathrm{a})^{4}(\mathrm{~b})^{0}+4(\mathrm{a})^{3}(\mathrm{~b})^{1}+6(\mathrm{a})^{2}(\mathrm{~b})^{2}+4(\mathrm{a})^{1}(\mathrm{~b})^{3}$ <br> $+1(\mathrm{a})^{0}(\mathrm{~b})^{4}$ |  |
| Notice: <br> - the exponents of a are descending while <br> - the exponents of $b$ are ascending, and <br> - the sum of the exponents is always the <br> given exponent in the binomial <br> expansion. |  |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(G) use the properties of logarithms to evaluate or transform logarithmic expressions;

Use the properties of logarithms.
Assume $x$ and $y$ are positive real numbers.
Properties of Logarithms:

1. $\log _{a} x y=\log _{a} x+\log _{a} b$
2. $\log _{a} \frac{x}{y}=\log _{a} a-\log _{a} b$
3. $\log _{a}(x)^{y}=y \log _{a} x$ for any real number $y$.

Properties of Natural Logarithms:

1. $\ln x y=\ln x+\ln y$
2. $\ln \frac{x}{y}=\ln x-\ln y$
3. $\ln x^{y}=y \ln x$ for any real number $y$.

## Example:

Expand the following:

$$
\log _{2}\left(\frac{3 x y}{2 z}\right)^{5}
$$

## Answer:

$$
5\left(\log _{2} 3+\log _{2} x+\log _{2} y-\left(\log _{2} 2+\log _{2} z\right)\right)
$$

Which can then be simplified to:

$$
5\left(\log _{2} 3+\log _{2} x+\log _{2} y-\log _{2} 2-\log _{2} z\right)
$$

You may distribute the 5 or not as you see fit.

Algebraic Reasoning
The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

| 5(H) generate and solve logarithmic equations in mathematical and real-world problems; | Example: |  |  |
| :---: | :---: | :---: | :---: |
|  | Example |  |  |
|  | Problem | If lime juice has a pH hydrogen ions (in mo hundredth? | 7 , what is the concentration of in lime juice, to the nearest |
|  |  | $\mathrm{pH}=-\log [\mathrm{H}+\mathrm{l}$ | Use the formula for pH . |
|  |  | $1.7=-\log x$ | Substitute the known pH into the formula, and represent $\mathrm{H}+$ with th variable $x$. |
|  |  | $-1.7=\log x$ | If $1.7=-\log x$, then $\log x=-1.7$. |
|  |  | $\begin{aligned} & x=10.1 .7 \\ & x=0.02 . \end{aligned}$ | Solve for $x$. |
|  | Answer | The concentration of hy | gen ions in lime juice is 0.02 . |

Example:

| Example |  |  |
| :---: | :---: | :---: |
| Problem | An earthquake is measured with a wave amplitude 392 times as great as $\mathrm{A}_{0}$. What is the magnitude of this earthquake using the Richter scale, to the nearest tenth? |  |
|  | $\begin{aligned} R & =\log \left(\frac{A}{A_{0}}\right) \\ R & =\log \left(\frac{392 A_{0}}{A_{0}}\right) \end{aligned}$ | Use the Richter scale equation. <br> Since A is 392 times as large as $A_{0}, A=392 A_{0}$. Substitute this expression in for A . |
|  | $\begin{gathered} R=\log 392 \\ R=2.5932 \ldots \\ R \cong 2.6 \end{gathered}$ | Simplify the expression $\left(\frac{392 A_{0}}{A_{0}}\right)=392$ <br> Use a calculator to evaluate the logarithm. |
| Answer The magnitude of this earthquake is 2.6 on the Richter scale. |  |  |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(I) generate and solve exponential equations in mathematical and real-world problems;

Example 1:
-

|  | Continuous Compound Interest: initial investment of $\$ 750$ interest rate of $10.5 \%$ <br> How long will it take it to double? <br> How much will there be after 10 years? <br> Answer 1: <br> Continuous compound interest: $\quad y=a e^{b x}$ $\begin{aligned} & \text { initial investment of } \$ 750 \\ & \text { interest rate of } 10.5 \%\end{aligned} \quad y=750 e^{.105 x}$ $b=.105$ <br> How long will it take it to double? $\begin{aligned} & \text { ill it take it to double? } \\ & \begin{array}{l} 1500=750 e^{.105 x} \\ 2=e^{.105 x} \\ \ln 2=\frac{\ln z}{.105} \\ \text { will there be after } 105 \end{array} \quad x=6.601 \mathrm{yrs} \end{aligned}$ <br> How much will there be after 10 years? $y=750 e^{.105(10)}$ <br> Example 2: <br> Radioactive Decay: <br> ${ }^{239} \mathrm{Pu}$ <br> Half-life of 24,100 <br> initial quantity 5 g <br> How much will there be after 1000 years? <br> Answer 2: |
| :---: | :---: |



| Algebraic Reasoning <br> The student uses process standards in mathem formulate models, and solve equations and inequa The student is expected to: | to evaluate expressions, describe patterns, ties using properties, procedures, or algorithms. |
| :---: | :---: |
| 5 (J) solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems; | Solve polynomials by factoring, using various methods of factoring such as grouping and/or synthetic division. This would also imply that students can use the remainder theorem and the rational roots theorem. <br> Example: <br> List the possible rational roots of $f(x)=$ $3 x^{3}-7 x^{2}-3 x-12$ <br> Example: <br> Given that a degree 5 polynomial with real coefficients has roots $3,2-i, 6+i$, what else must also be a root? <br> Example: <br> Find all roots of $y=2 x^{3}-5 x^{2}+12 x-30$ |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

| $\mathbf{5 ( K )}$ solve polynomial inequalities with real <br> coefficients by applying a variety of <br> techniques and write the solution set of the <br> polynomial inequality in interval notation in <br> mathematical and real-world problems; | Solve polynomial inequalities by graphing, <br> sign patterns and any other methods. |
| :--- | :--- |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

| 5(L) solve rational inequalities with real <br> coefficients by applying a variety of <br> techniques and write the solution set of the <br> rational inequality in interval notation in <br> mathematical and real-world problems; | Solve rational inequalities by graphing and <br> using sign patterns. <br> Example <br> Solve the rational inequality $\frac{x-3}{x+1} \leq 0$ |
| :--- | :--- |
| Solution by graphing: <br> NORMAL FLOAT AUTO REAL RADIAN MP <br> CaLC INTERSECT |  |


|  |  <br> Either way, the solution is ( $-1,3$ ]. |
| :---: | :---: |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

| 5(M) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and | Use trig identities to simplify expressions. <br> Reciprocal Identities $\begin{array}{lll} \sin u=\frac{1}{\csc u} & \cos u=\frac{1}{\sec u} & \tan u=\frac{1}{\cot u} \\ \csc u=\frac{1}{\sin u} & \sec u=\frac{1}{\cos u} & \cot u=\frac{1}{\tan u} \end{array}$ |
| :---: | :---: |
|  | Quotient Identities $\tan u=\frac{\sin u}{\cos u} \quad \cot u=\frac{\cos u}{\sin u}$ |
|  | Pythagorean Identities <br> $\sin ^{2} u+\cos ^{2} u=1 \quad 1+\tan ^{2} u=\sec ^{2} u \quad 1+\cot ^{2} u=\csc ^{2} u$ |
|  | Cofunction Identities $\begin{array}{ll} \sin \left(\frac{\pi}{2}-u\right)=\cos u & \cos \left(\frac{\pi}{2}-u\right)=\sin u \\ \tan \left(\frac{\pi}{2}-u\right)=\cot u & \cot \left(\frac{\pi}{2}-u\right)=\tan u \\ \sec \left(\frac{\pi}{2}-u\right)=\csc u & \csc \left(\frac{\pi}{2}-u\right)=\sec u \end{array}$ |
|  | Even/Odd Identities $\begin{array}{lll} \sin (-u)=-\sin u & \cos (-u)=\cos u & \tan (-u)=-\tan u \\ \csc (-u)=-\csc u & \sec (-u)=\sec u & \cot (-u)=-\cot u \end{array}$ |


|  | Sum and Difference Formulas (See the $\begin{aligned} & \sin (u+v)=\sin u \cos v+\cos u \sin v \\ & \sin (u-v)=\sin u \cos v-\cos u \sin v \\ & \cos (u+v)=\cos u \cos v-\sin u \sin v \\ & \cos (u-v)=\cos u \cos v+\sin u \sin v \end{aligned}$ <br> Example <br> Simplify $\sin x \cos ^{2} x-\sin x$. <br> Solution: $\begin{aligned} \sin x \cos ^{2} x-\sin x & =\sin x\left(\cos ^{2} x-1\right) \\ & =-\sin x\left(1-\cos ^{2} x\right) \\ & =-\sin x\left(\sin ^{2} x\right) \\ & =-\sin ^{3} x \end{aligned}$ <br> Example <br> Simplify $\sin t+\cot t \cos t$. <br> Solution: $\begin{aligned} \sin t+\cot t \cos t & =\sin t+\left(\frac{\cos t}{\sin t}\right) \cos t \\ & =\frac{\sin ^{2} t+\cos ^{2} t}{\sin t} \\ & =\frac{1}{\sin t}=\csc t \end{aligned}$ |
| :---: | :---: |

## Algebraic Reasoning

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

5(N) generate and solve trigonometric equations in mathematical and real-world problems.

Use trig identities to generate trig equations to model situations.

## Example

The Ferris wheel at the country fair has a diameter of 120 feet. Joseph reaches the top of the ride for the first time 15 seconds after the Ferris wheel starts. He times it and finds that it takes him 50 seconds to complete one revolution.

|  | Before he got on, he noticed that the platform where the ride starts is 10 feet from the ground. <br> Solution: $y=\cos \left(\frac{2 \pi}{25}(x-15)\right)+70$ <br> Example <br> The The tide, or depth of the ocean near the shore changes throughout the day. The depth at a certain bay can be modeled by $d=$ $45-34 \cos \frac{\pi}{6.2} t$ where $d$ is the depth of the water in feet and t is the time in hours. Consider a day where $t=0$ represents 12:00AM. For that day, when do the high and low tides occur? <br> Solution: The first high tide occurs at 6:20 am and the first low tide occurs at 12:40 pm <br> Solve trig equations over the real numbers and over given intervals such as $[0,2 \pi)$. <br> Example 1 <br> Solve $2 \sin x-1=0$. <br> Solution: $\begin{aligned} 2 \sin x-1 & =0 \\ 2 \sin x & =1 \\ \sin x & =\frac{1}{2} \\ x=\frac{\pi}{6}+2 n \pi & \text { or } \quad x=\frac{5 \pi}{6}+2 n \pi \end{aligned}$ <br> Example 2 <br> Solve $4 \tan ^{2} x-1=0$ in the interval $0 \leq x<2 \pi$. Solution: |
| :---: | :---: |


|  | $\begin{aligned} & 4 \tan ^{2} x-1=0 \\ & 4 \tan ^{2} x=1 \\ & \tan ^{2} x=\frac{1}{4} \\ & \tan x= \pm \frac{1}{2} \\ & x \approx 0.464+n \pi \end{aligned}$ <br> or $x \approx-0.464+n \pi$ <br> where $n$ is any integer. The solutions that are in the interval $0 \leq x<2 \pi$ are: $\begin{array}{ll} x \approx 0.464 & x \approx 0.464+\pi \approx 3.61 \\ x \approx-0.464+\pi \approx 2.628 & x \approx-0.464+2 \pi \approx 5.82 \end{array}$ <br> Example: <br> Solve $2 \sin ^{2} x+3 \cos x-3=0$. <br> Solution: $\begin{aligned} & 2 \sin ^{2} x+3 \cos x-3=0 \\ & 2\left(1-\cos ^{2} x\right)+3 \cos x-3=0 \\ & 2 \cos ^{2} x-3 \cos x+1=0 \\ &(2 \cos x-1)(\cos x-1)=0 \\ & x=2 n \pi, \quad x=\frac{\pi}{3}+2 n \pi, \quad x=\frac{5 \pi}{3}+2 n \pi \end{aligned}$ |
| :---: | :---: |

