Geometry Texas Mathematics: Unpacked Content

What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?

Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

Geometry Primary Focal Areas:

The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to

create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

(4) These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

For all Process Standards: It should be a continuing learning process throughout the course.

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(A) apply mathematics to problems arising in everyday life, society, and the workplace;	Methods & concepts should all have real world application problems

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;	Students should be introduced to and encouraged to use a 5-step problem-solving process.

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;	Select appropriate tools and use the most reasonable technique to solve problems

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;	Students need to express mathematical ideas in variety of formats.

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(E) create and use representations to organize, record, and communicate mathematical ideas;	Create, Use Big Idea: Students should be able to create their own visual representation (construction, table, diagram, picture) to model a problem or add information to the given visual representation to break down the problem. Once students find the solution, they should be able to effectively explain their process for finding the solution.

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(F) analyze mathematical relationships to connect and communicate mathematical	Analyze, Communicate Big Idea:

ideas; and	Students should be able to make connections between the content and its applications. Mastery needs to be shown through written or verbal explanations

Mathematical Process Standards	**Unpacking** What does this standard mean that a student will know and be able to do?
1(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	Display, Explain, Justify Big Idea: Students should be able to have mathematical discussions while referring to the original problem and show understanding through logical arguments.

Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
2(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;	A big idea for this student expectation (SE) is that there are an infinite amount of points on a line between two endpoints and students will need to find any point that is a fractional distance away. One-dimensional: For any fractional distance, students need to multiply the difference of one coordinate by the fractional distance (k).

	Students will need to use this formula to find any fractional distance: $(x_2 - x_1) * k + x_1$ Examples: On a number line, point A is located at -3 and point B is located at 21. Find points that are located one-fourth, one-third, three-fifths, etc. of the distance between the endpoints. Two-dimensional: For any other fractional distance, students need to multiply the difference of each set of coordinates by the fractional distance (k) and then add that distance to the original coordinates (x_1, y_1). Students will need to use this new formula to find any fractional distance: $((x_2 - x_1) * k + x_1, (y_2 - y_1) * k + y_1)$ Instead of using this to find only the midpoint: $((x_1 + x_2) / 2, (y_1 + y_2) / 2)$ Examples: Point P has coordinates (-8, 5) and Point Q has coordinates (4, -1). Find points that are located one-fourth, one-thrid, three-fifths, etc. of the distance between the endpoints. Teachers should initially make a connection between the midpoint formula and general form which can apply to any ratio. Example Reference: http://www.tpub.com/math2/2.htm

	Misconceptions: If you multiply by the ratio(r) be sure to distinguish from radius(r) and ratio(r).

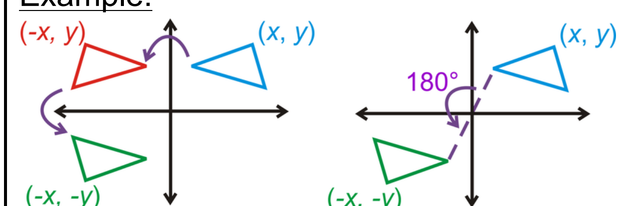
Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
2(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and	<u>Prior Knowledge:</u> Students have found distance on the coordinate grid using the Pythagorean theorem in 8th grade. Students have a concept of slope from Algebra I. Students will have seen the average (mean) since 6th grade, which is used to find the midpoint. The coordinate grid can be used to derive all the formulas. 1. Students need to be shown how to take the Pythagorean Theorem and derive the distance formula. Students should be able to apply the formula to solve for missing lengths and then using that information to determine relationships (including determining congruence of segments). Teachers need to explain the importance of substituting in consistent order in the formula. 2. Teachers should review the concept of slope (change in y's over change in x's), how to find slope, parallel and perpendicular slopes. (**NOTE:** It should be emphasized perpendicular is opposite reciprocal vs. negative reciprocal. It should also be emphasized parallel lines or segments have the same slope.)

	Given a geometric figure on a grid, students should be able to identify the slope of each side. Once they have identified the slopes of each side, students should be able to determine if there are any parallel or perpendicular sides. 3. Teachers should explain the connection of the midpoint formula to finding an average. Students should be able to use the midpoint formula. (**NOTE**: A problem could give the midpoint and the coordinates of one endpoint, and then ask students to find the coordinates of the other endpoint.) Once students have found the midpoint, they are able to verify medians, midsegments, and perpendicular bisectors. <u>Example Problem (Covers all parts of TEK)</u> 1. Students plot four points on a grid to create a quadrilateral. Students would need to use the distance formula to find side lengths and find the slope of each side to prove it is or is not a square.

Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
2(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.	Prior Knowledge: Students write an equation of a line that goes through a given point that is parallel or perpendicular to another line (Algebra 2E, 2F, 2G). Teachers will still need to review the topic (point slope formula, slope intercept formula) even though it was covered in Algebra. Teachers will need to show how this will be applied to figures on a coordinate grid. Examples: (1) A triangle with endpoints is given on a coordinate grid and students need to find an equation of the altitude for one of the sides. (2) Find the equation of the tangent line to a circle given the endpoint of the radius on the coordinate grid. (3) Given Rectangle ABCD on a grid, find the equation for side AB, given the slope of side CD is ½.

Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
3(A) describe and perform transformations of figures in a plane using coordinate notation;	Students will need to explain how the transformation is changing the x and y for each point on the figure. Students will need to do a transformation on the coordinate grid.

	NOTES Translation - orientation does not change, can be a composition of two reflections across parallel lines Rotation - can be a composition of reflections across intersecting lines Teacher should address clockwise vs. counter clockwise rotation. Reflection - identify line of reflection Can be any line in coordinate plane, not just the x and y axis Dilation - center can be anywhere on plane <u>Example Coordinate Notation:</u> Translation (x, y) → (x +1, y - 4) Reflection (x, y) → (-x, y); (x, y) → (x, -y); etc. Rotation of 90 degrees around origin: (x, y) → (y, -x); Rotation of 180 degrees around origin: (x, y) → (-x, -y); Rotation of 270 degrees around origin: (x, y) → (-y, x); Rotation of 360 degrees around origin: (x, y) → (x, y) Dilation (x, y) → (2x, 2y) (notations are applicable to clockwise rotations.)

Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
3(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;	**Rigid transformations** include rotation, reflection, and translation (size/shape does not change but orientation does) **Non-rigid transformations** would include dilation. **Composition of Transformations:** Combining two or more transformations. Be aware that the center of dilation can be in point on the coordinate plane. Find the new image or the original image going through a series of transformation (rigid, non-rigid or a combination of both) Misconceptions: Remind students about the uses of the apostrophe for notation Dilation - center of dilation is NOT always the origin <u>Example:</u> center of dilation **not** (0,0) Determine a rule ((x,y) → (?,?) that can be use to dilate a figure on the coordinate grid by a scale factor of ½ using (2,4) as the center of dilation. ((½ (x - 2) + 2, ½ (y - 4) + 4) = (½ x + 1, 1/2y +2) <u>Example:</u> 

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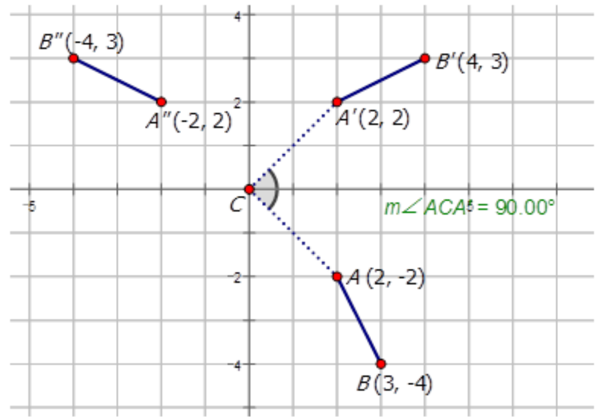
<tr><td></td><td>

First line AB is rotated about the origin by $90°$CCW. Then the line $A'B'$ is reflected about the y-axis to produce line $A''B''$.</td></tr>
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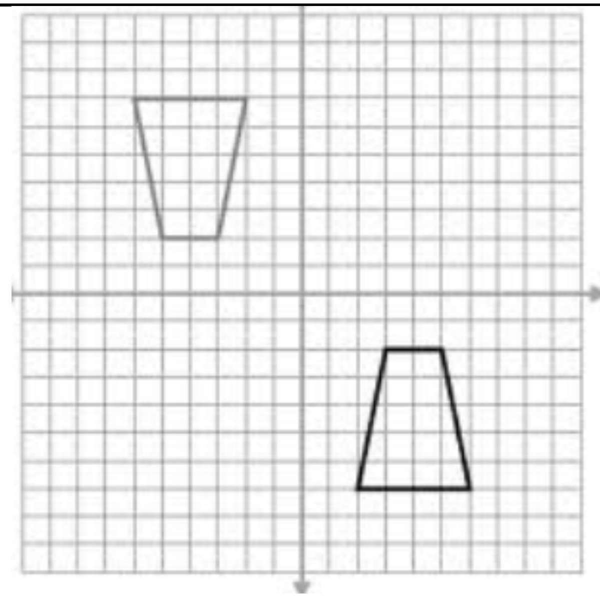
Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
3(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and	Students should be able to identify the process and the order of how the image was changed. Teachers need to show examples on the coordinate plane and off the plane. Off the plane could be on patty paper and real world examples. Off Coordinate Plane - Not limited to graphs How It Could Be Assessed: Given pre image & image - students must list the compositions applied

	Misconception: Students assume that performing the transformations in any order will result in same result

Coordinate and Transformational Geometry	**Unpacking** What does this standard mean that a student will know and be able to do?
3(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.	**Note to Emphasize** Reflectional - over a line Rotational - in respect to a point <ins>Note To Teacher:</ins> Use a variety of figures when you introduce the concept to students Horizontal Reflection Symmetry Vertical Reflection Symmetry <ins>Misconception:</ins> 180 degree rotation can also be a reflection, but many times it is not. This example is a rotation of 180 degrees but not a reflection across the line y = x. This can be misleading for students especially with isosceles triangles. Make sure to have students label their vertices.

Logical Argument and Constructions	Unpacking What does this standard mean that a student will know and be able to do?
4(A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;	Teachers need to just explain definitions to students. **Undefined terms** - term or word that doesn't require further explanation or description. It already exists in its most basic form. Euclidean Geo - Point, line, plane

	Definitions - classifies and gives critical attributes **Postulates** - A statement, also known as an axiom, which is taken to be true without proof **Conjectures**- statement believed to be true based on inductive reasoning (process of reasoning that a rule or statement is true based on pattern) **Theorems** - statement that has been proven on the basis of previously established statements, such as other theorems, and previously accepted statements, such as postulates.

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
4(B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;	**Big Idea:** Students need to determine the truth values for each statement, starting with the original conditional statements **Conditional statement:** If p then q, (p is the hypothesis and q is the conclusion.) **Testing validity:** A conditional statement is false only when the hypothesis, p, is true and the conclusion, q, is false. **Converse:** If q then p

	Inverse: If not p then not q **Contraprositive:** if not q, then not p **Biconditional Statement:** p if and only if (iff) q A biconditional statement exists only when the original statement and its converse are both true. <u>Students Need to Know:</u> In order to write a biconditional statement, then the conditional and converse must both be true. *Need to verify both are true in the truth table before you can write the biconditional statement Example: Given the following conditional statement: “If an angle is 90 degrees then the angle is a right angle.” This statement is true. If we take the converse which reads, “If an angle is a right angle, then the angle is 90 degrees.” Since the converse is true as well, we can now write a biconditional statement which will read, “An angle is 90 degrees iff the angle is a right angle.”

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
4(C) verify that a conjecture is false using a counterexample; and	A conjecture is an if- then statement that is false when the hypothesis, p, is true and the conclusion, q, is false **Examples:** The statement "If a figure is a triangle then all angles are congruent." is false because a counterexample would be a 30-60-90 triangle. **Notes:** Teachers need to be aware that sometimes there will be multiple counterexamples, one counterexample, or no counterexamples. An extension of counterexample is determining if a statement is sometimes true, always true, or never true based on the number of counterexamples. **Misconception:** Counterexamples must satisfy the hypothesis of the conditional statement, or they cannot be considered as counterexamples. Example: Conditional: If an angle is acute, then it is less than 180 degrees. Some students will say a counterexample could be 120 degrees. However, this cannot be a counterexample because 120 degrees does not satisfy the hypothesis of the statement, and therefore, would not be considered.

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
4(D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.	**Teachers:** In order to illustrate the existence of different mathematical geometries, euclidean and spherical geometries can be investigated concretely. **Euclidean geometry:** geometry based on Euclid's geometry -Parallel lines do not intersect and are in one plane -The sum of the angles of a triangle is 180 degrees **Spherical Geometry:** the geometry of figures on a sphere -Parallelism-circular routes that do not intersect (i.e. latitudes) -The sum of the angles of a triangle is between 180 and 540 degrees. Note: Globe beach balls may be helpful to teach the concepts of spherical geometry.

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
5(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and	<u>Teachers should:</u> Show the relationships of parallel lines cut by a transversal & patterns it creates How patterns lead to theorems

angles of circles choosing from a variety of tools;

Example: Pattern in interior angle sums

Number of Sides	Polygon Name	Number of Triangles Formed	Interior Angle Sum Measure
3	Triangle	1	180°
4	Quadrilateral	2	360°
5	Pentagon	3	540°
6	Hexagon	4	720°
7	Heptagon	5	900°
⋮	⋮	⋮	⋮
n	n - gon	$(n-2)$	$(n-2)\cdot 180°$

<u>Students should:</u>

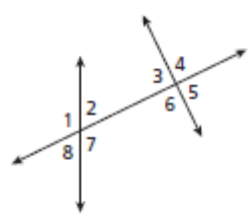
Investigate patterns

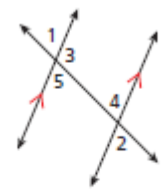
Discover theorems based on investigation type activities

<u>Misconceptions:</u>

Angle relationships exist with any two lines at transversal. Students must understand that lines cut by a transversal must be parallel of the angles relationships to exist.

Examples:



	Angles 8 and 4 are Alternate exterior angles that are NOT congruent. VS.  Angles 1 and 2 are Alternate exterior angles that ARE congruent BECAUSE the two lines are parallel.

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
5(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;	<u>Teachers should:</u> Make formal geometric constructions with a variety of tools and methods Possible tools - compass,folding, patty paper, string, reflective mirrors, geo software <u>Students should:</u> Copy a segment Copy an angle Bisect an angle

Bisect an angle

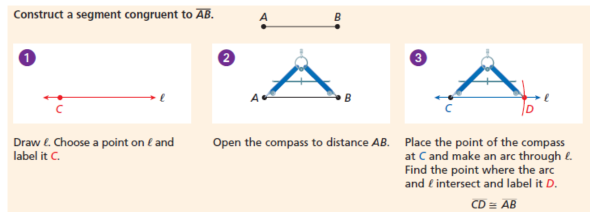
Construct perpendicular lines including perpendicular bisectors

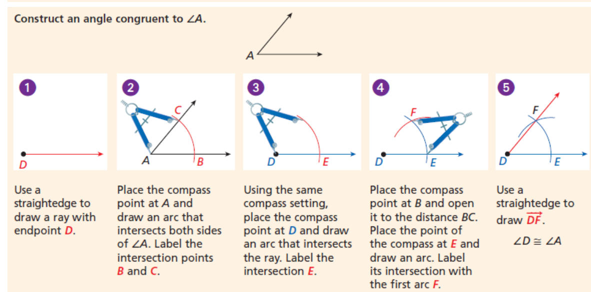
Construct parallel line

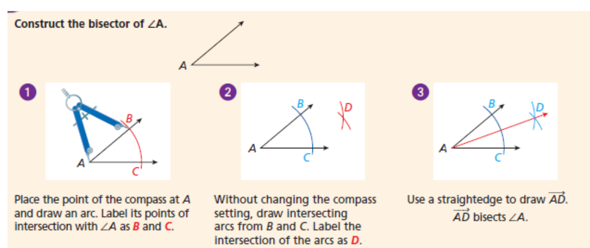
Misconceptions:

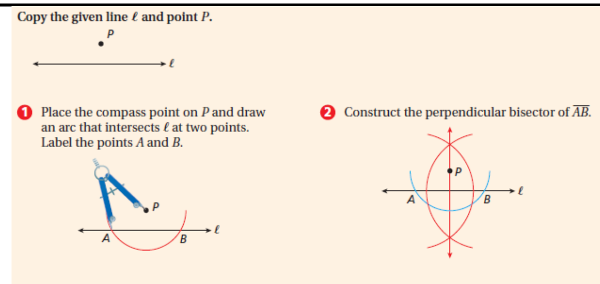
Assume one part is true (ex. 90 degrees)

Examples:







Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
5(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and	 <u>Students:</u> Prove theorems about lines & angles Vertical angles are congruent Alternate interior & corresponding are congruent Endpoints of a line segment that is cut by a perpendicular bisector are equidistant from the point of intersection.

Logical Argument and Constructions	**Unpacking** What does this standard mean that a student will know and be able to do?
5(D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.	 <u>Prove:</u> The triangle inequality theorem states that any side of a triangle is always

shorter than the sum of the other two sides.

Converse: A triangle cannot be constructed from three line segments if any of them is longer than the sum of the other two.

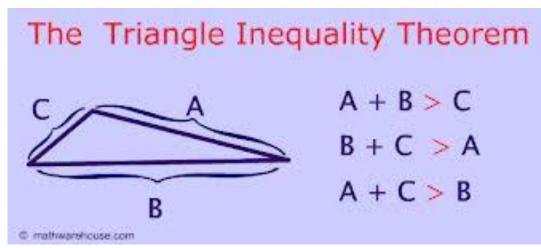
Examples:

1. Which of the following could represent the lengths of the sides of a triangle ?   Choose one:

1, 2, 3

6, 8, 15

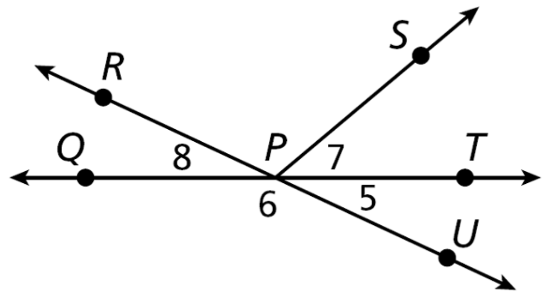
5, 7, 9



Proof and Congruence	**Unpacking** What does this standard mean that a student will know and be able to do?
6(A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;	Confirm theorems about angles formed by Vertical lines, parallels cut by transversal Prove equidistance b/t endpoints of a segment & points on its perpendicular bisector Apply these relationships to solve problems Students should know how to: o Prove vertical angles are congruent o Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent. o Find midpoint and possibly use distance formula. o Understand labels & symbols in order to apply the relationships. o Finding missing angles and measurements. o Identifying if lines are parallel o Every point on a point lying on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. Teachers should show: Vertical Angle Theorem Corresponding Angles Perpendicular Transversal Perpendicular Bisector Theorem

Parallel Transversal (4) - Alt Int/ Ext, SS Int/Ext
Examples on how to solve missing angles

Misconceptions
Vocabulary, proofs
Vertical angles are formed by 2 straight lines, not simply rays that come together at the same point. For example:



Angle RPS is not a vertical angle to Angle QPU because Q and S do not lie along the same line.

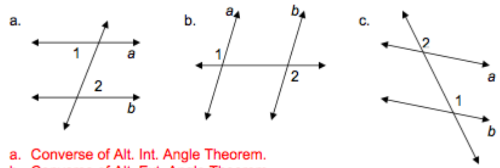
Example w/ proving angle measurements

GIVEN: Line *a* is parallel to line *b*.
PROVE: $\angle 1 \cong \angle 3$

Statement	Justification
Line *a* is parallel to line *b*.	Given
$\angle 1 \cong \angle 2$	Corresponding Angle Postulate
$\angle 2 \cong \angle 3$	Vertical Angle Theorem
$\angle 1 \cong \angle 3$	Substitution (or, transitive)

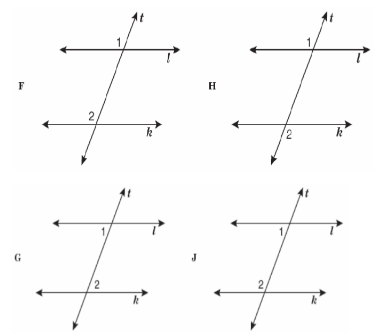
Example proving theorems

1. State the postulate or theorem that allows the conclusion that if $\angle 1 \cong \angle 2$, then $a \parallel b$.

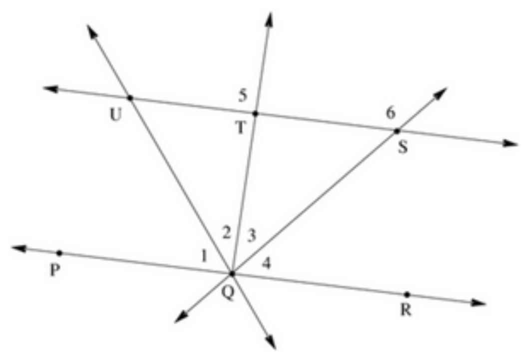


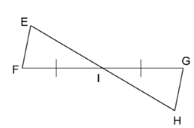
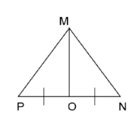
a. Converse of Alt. Int. Angle Theorem.
b. Converse of Alt. Ext. Angle Theorem.
c. Converse of Corresponding Angle Postulate.

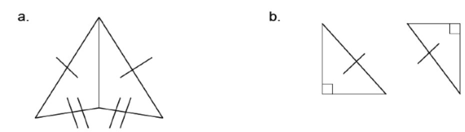
50 Mitch drew lines *l*, *k*, and *t*. Lines *l* and *k* were parallel to each other. Mitch measured 2 alternate interior angles. He labeled the angles 1 and 2. Which of the following shows angles 1 and 2 correctly labeled?



Example Equidistance with perpendicular bisector



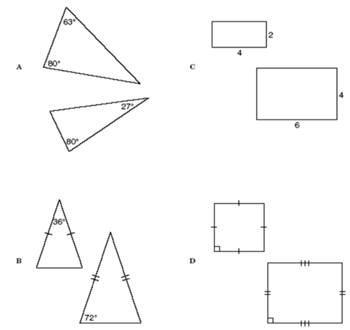
Proof and Congruence	**Unpacking** What does this standard mean that a student will know and be able to do?
6(B) prove two triangles are congruent by applying the Side-Angle-Side(SAS), Angle-Side-Angle(ASA), Side-Side-Side(SSS), Angle-Angle-Side(AAS), and Hypotenuse-Leg congruence conditions;	Big Idea - Use the definition of congruence to develop and explain the triangle congruence criteria; ASA, SSS, and SAS. <u>Teachers should:</u> For Right Triangles: Explain postulates including Leg-Leg (SAS), Hypotenuse-Angle (AAS) **Reminder: “A’ is not the right angle, it must be one one of the acute angles Hypotenuse-Leg (SSS) - only true for right triangle bc since it has a right angle you can use the pythagorean theorem. it only will give you one possible value for the 3rd side. Creates SSS congruence Review special segments Give examples proving congruence <u>Students should:</u> Be able to prove two triangles congruent using theorems. <u>Examples</u> Give any additional information that would be needed to prove the triangles congruent by the method given.   Determine which method can be used to prove the triangles congruent from

	the information given. For some pairs, it may not be possible to prove the triangles congruent. For these, explain what other information would be  needed to prove congruence. Misconception: Part a - Students forget that a shared side b/t two figures have the same length bc it is the same side. Students mistakenly use Angle-side-side as a congruence theorem. HL - is not a version of SSA because you cannot use the 90 degree angle as your ‘A’

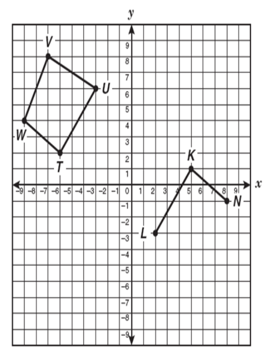
Proof and Congruence	**Unpacking** What does this standard mean that a student will know and be able to do?
6(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;	Big Idea - identify congruent figures and identify their corresponding parts Rigid Transformation - when the size of the figure does NOT change (translation, rotation, reflection, NOT dilation)

	<u>Teachers should show:</u> Explain what a rigid transformation is and how the size of the figure will not change Explain definition of congruence & corresponding parts Relate terminology of rigid transformations (reflection, translation, rotation) to congruent transformations CPCTC **Review how to find the length of a segment in the coordinate plane, slope of parallel/perpendicular lines <u>Students should do:</u> Look at two figures & identify corresponding parts Prove congruence based on congruent corresponding parts <u>Misconceptions:</u> Students should understand that figures can be congruent even when they are rotated or flipped. Congruence is true even if figures have different orientation. Examples:

<table>
<tr><td></td><td>
Use the information in each diagram to find the pair of similar polygons.



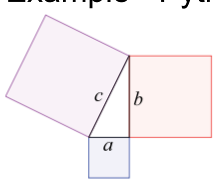
7 The graph shows quadrilateral $TUVW$, $\overline{KN}$, and $\overline{KL}$.



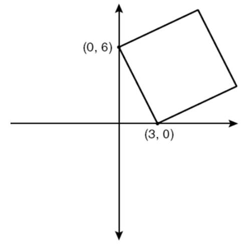
At what coordinate point should vertex M be placed to make quadrilateral $KLMN$ congruent to quadrilateral $TUVW$?

A (6, −4)
B (4, −5)
C (5, −5)
D (6, −5)
</td></tr>
</table>

Proof and Congruence	**Unpacking** What does this standard mean that a student will know and be able to do?
6(D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians	<u>Big Idea</u> - verify theorems about relationships in TRIANGLES and apply to solve problems <u>Teaches need to show:</u> Proof Pythagorean theorem - need to show an Algebraic proof and concrete

and apply these relationships to solve problems	proof Sum of interior angles - (180) Base angles of isosceles - base angles are congruent M**idsegment** of a triangle is a segment connecting the midpoints of two sides of a triangle. This segment has two special properties. It is always parallel to the third side, and the length of the midsegment is half the length of the third side. **Median** of a triangle is a line segment that extends from one vertex of a triangle to the midpoint of the opposite side. All 3 will intersect to form centroid. THe centroid splits the medium in a two to one ratio. <u>Students need to:</u> Understand relationships in theorems in order to apply and solve problems Find midpoint of a side, midsegment, median Find length midsegment given parallel side Apply Pythagorean formula Find missing angles knowing angle sum is 180 or find missing angle given a single base angle Example - Pythagorean proof  Pythagorean Theorem Application

What is the area of the square in the figure below?



A 5.2 square units

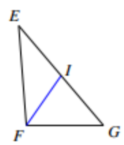
B 6.7 square units

C 27 square units

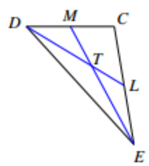
D 45 square units

Median Application

Find x if $GE = 3x + 5$ and $IE = x + 6$



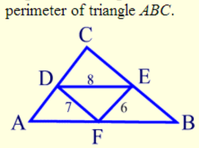
Find x if $ET = 3x + 2$ and $EM = 5x$



$\frac{2}{3}(EM)=ET$

Midsegment Examples

2. Given DE, DF, and FE are the lengths of mid-segments. Find the perimeter of triangle ABC.



Solution:
The mid-segment is half of the third side.
6 is half of 12 so $AC = 12$
7 is half of 14 so $CB = 14$
8 is half of 16 so $AB = 16$
The perimeter of the large triangle ABC is:
$12 + 14 + 16 =$ **42.**

Proof and Congruence	**Unpacking** What does this standard mean that a student will know and be able to do?
6(E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.	Big Idea - prove what type of quadrilateral and apply to solve problems Parallelogram – quad with two pair of parallel sides § Opposite sides are parallel § Opposite sides are congruent § Opposite angles are congruent § Consecutive angles are supplementary § Diagonals of a parallelogram bisect each other Unique Characteristics o Rectangle – parallelogram with four right angles § Diagonals of a rectangle are congruent o Rhombus – parallelogram with four congruent sides § Diagonals of a rhombus are perpendicular to each other and angle bisectors o Square – parallelogram with four right angles and four congruent sides § Diagonals are angle bisectors <u>Teachers should show:</u> Explain the characteristics Proving congruent sides Finding angle measurements

	Students need to: Solve for all missing parts of the figure Find side measurements using distance formula Find slope of each side & diagonals to identify perpendicular angles & parallel sides Find diagonal length Identify the figure given based on key attributes Misconceptions Students will try to classify the figure as only one of the different options versus recognizing the overlap. For example, a rhombus is a square, rectangle and parallelogram Students many times think parallelograms have a line of symmetry and therefore try to mark angles congruent that are not congruent. Examples 1. In the parallelogram below, PG = 2x – 7, MR = x + 5, and MG = 2x – 5. Find the value of x, PG, MR, and MG.

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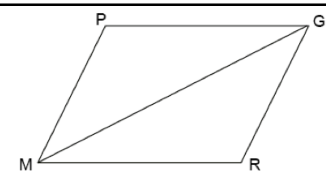
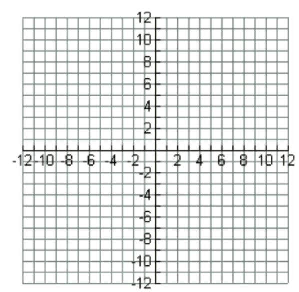


Draw figure ABCD using the following ordered pairs: A(0, 0), B(5, 5), C(6, 12), and D(1, 7). Complete the table below. How do you know this is a rectangle versus a square?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</table>

<table>
<tr>
<td>Similarity, Proof, and Trigonometry</td>
<td>Unpacking
What does this standard mean that a student will know and be able to do?</td>
</tr>
<tr>
<td>7(A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and</td>
<td>Big Idea - identify similar figures, their proportional sides and congruent angles

Prior Knowledge - refer back to dilations covered through transformation (w/ any point as center of dilation), similar figures

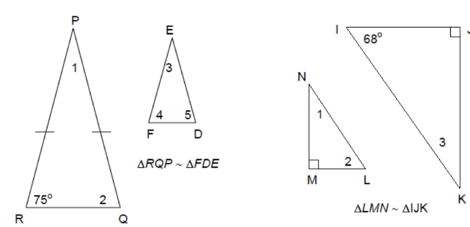
<u>Teachers should show:</u></td>
</tr>
</table>

Characteristics of similar figures
Using proportions to find missing sides (Cross products are equal)
Identifying corresponding angles

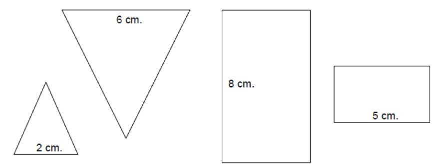
Students should:
Know key attributes similar figures (congruent angles & corresponding sides are proportional)
Find missing angles and side lengths
Know how to apply a scale factor (> 1 and <1)

Misconceptions:
When naming figures order matters. Corresponding angles should be in the same order in a figure.

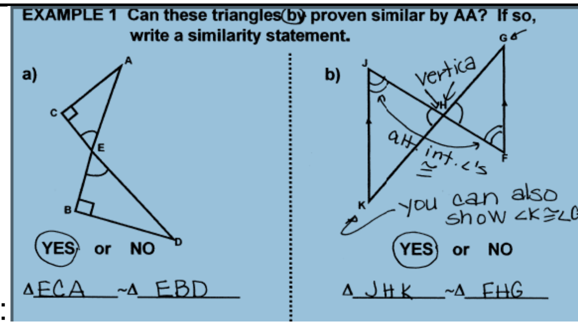
Examples:
Use properties of triangles and the similarity statement to find the measures of the numbered angles.



Using the given measurements, find the scale factor (similarity ratio) of each of the following sets of similar figures, comparing smaller to larger; then comparing larger to smaller.

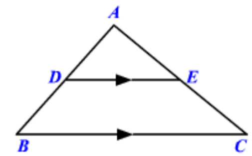
	

Similarity, Proof, and Trigonometry	**Unpacking** What does this standard mean that a student will know and be able to do?
7(B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.	Big Idea - if triangles have two congruent angles, then they are similar triangles therefore sides are proportional. <u>Teacher should:</u> Show example of similar triangles with AA Show examples where congruent angles might be from vertical angles, parallels cut by a transversal (prior knowledge) <u>Students should:</u> Identify corresponding angles Understand the AA theorem Similar triangles - proportional sides Set up proportions and solve for missing sides Find the 3rd angle for both triangles

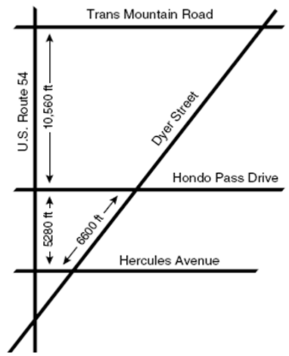
<table>
<tr><td></td><td>Examples: 

Misconceptions
The sides in SSS and SAS must be proportional now (and not congruent like they were with proving triangles congruent)</td></tr>
</table>

Similarity, Proof, and Trigonometry	**Unpacking** What does this standard mean that a student will know and be able to do?
8(A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and	Big Idea - prove theorems & apply to solve Teachers should: Complete proofs of AA Similarity Postulate, SSS Similarity Theorem, SAS Similarity Theorem, and the Triangle Proportionality Theorem. Triangle Proportionality Theorem If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally

	 If $\overline{DE} \parallel \overline{BC}$, then $\frac{AD}{DB} = \frac{AE}{EC}$ <u>students need to:</u> Complete proofs of AA Similarity Postulate, SSS Similarity Theorem, SAS Similarity Theorem, and the Triangle Proportionality Theorem. Find missing sides and angles Misconceptions Must have the included angle to be true for SAS The sides in SSS and SAS must be proportional now (and not congruent like they were with proving triangles congruent) Examples

In El Paso, Texas, the streets named Hercules Avenue, Hondo Pass Drive, and Trans Mountain Road are parallel. They all intersect Dyer Street and U.S. Route 54, as shown on the map below.



If all of these streets are straight line segments, how long is Dyer Street between Hercules Avenue and Trans Mountain Road?

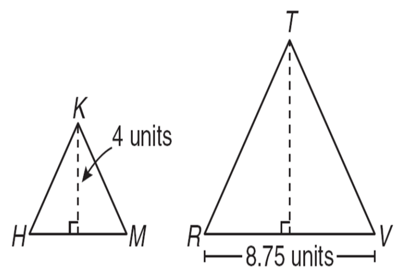
A 8,450 ft

B 9,900 ft

C 13,200 ft

D 19,800 ft

51 In the figures below, $\Delta HKM \sim \Delta RTV$, and the area of ΔHKM is equal to 10 square units.



What is the area of ΔRTV?

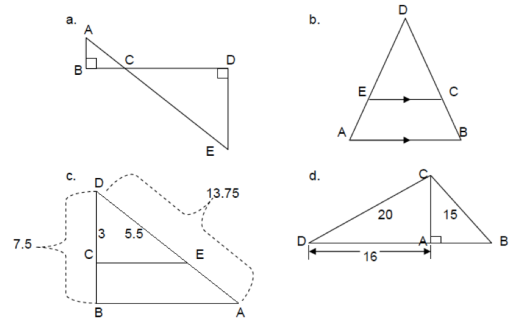
A 30.625 square units

B 87.5 square units

C 21.875 square units

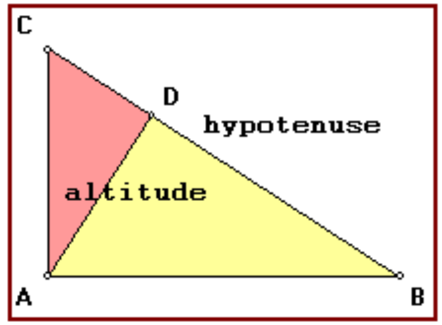
D 61.25 square units

Give the appropriate theorem or postulate that can be used to prove the triangles similar and write the similarity statement.

Similarity, Proof, and Trigonometry	**Unpacking** What does this standard mean that a student will know and be able to do?
8(B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.	Big Idea - identify and apply relationships of an altitude drawn to the hypotenuse to solve problems Teachers need to Show: *Geometric Mean - It is the nth root of the product of n numbers. That means you multiply the numbers together, and then take the nth root, where n is the number of values you just multiplied.* Altitude on a Right Triangle- The measure of the **altitude** drawn f**rom the vertex of the right angle** of a right triangle to its hypotenuse is the **geometric mean between the measures of the two segments of the hypotenuse**. In terms of our triangle, this theorem simply states what we have already shown: $AD = \sqrt{CD \cdot DB}$ since AD is the altitude drawn from the right angle of our right triangle to its

hypotenuse, and CD and DB are the two segments of the hypotenuse.



<u>Students Need To:</u>

Be able to calculate geometric mean

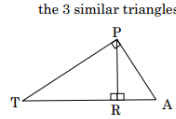
Work backwards from geo mean to a side

Draw the altitude

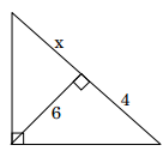
Identify similar triangles created

Examples:

1) If an altitude is drawn to the hypotenuse of triangle TAP below, then name and redraw the 3 similar triangles created.

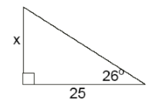


2)

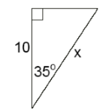


Similarity, Proof, and Trigonometry	**Unpacking** What does this standard mean that a student will know and be able to do?
9(A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and	Big Idea - trig ratios to solve for missing parts <u>Teacher should:</u> Show how to set up a trig ratio Explain vocabulary hypotenuse, opposite & adjacent How to solve with the calculator Explain inverse calculation for angles (on calculator) <u>Students should be able to</u> A.) correctly identify the hypotenuse, adjacent and opposite sides of a triangle in relation to a given angle. B.) Use inverse operations to calculate a missing angle of a right triangle given two sides. <u>Example Problems</u>

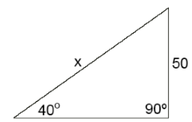
a.



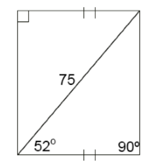
b.



c.



d.



Congruent sides are also parallel.
Perimeter of quadrilateral = ________

<u>Example Word Problems</u>

A new rope must be ordered for the flagpole in front of the school. Before ordering the rope, the height of the pole must be determined. It is observed that the flagpole casts a shadow 10.5 meters long when the sun is at an angle of elevation of 33 degrees. How tall is the flagpole?

A cat is trapped on a tree branch 18.5 feet above the ground. The ladder is only 20 feet long. If you place the ladder's tip on the branch, what angle must the ladder make with the ground?

Misconceptions:

When to use the sine, cosine, and tangent keys versus the inverse keys. (Only use the inverse key if solving for an angle.)

Similarity, Proof, and Trigonometry	**Unpacking** What does this standard mean that a student will know and be able to do?
9(B) apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.	Big Idea - applying special right triangles and Pythagorean theorem & triples to solve for missing sides <u>Teacher should show:</u> How to identify the sides of a special right triangle -Show how 30-60-90 is an equilateral cut in half -Derive side lengths of 30-60-90 & 45-45-90 using basic triangle starting with one as a side length -Use that triangle to create relationships $x;\ x;\ x\sqrt{2}$ $x;\ 2x;\ x\sqrt{3}$ - simplify the radical and rationalize the denominator How to use pythagorean triples and their multiples -Focus on identifying triples such as 3,4,5 & 5, 12, 13 (as well as dilations) Also show 7, 24, 25 & 8, 15, 17 <u>Students should:</u> Have previous knowledge of simpllifying radicals, multiplying/dividing radicals and rationalizing denominators. Use ratios, special relationships, pythagorean theorem to solve for missing sides Complete multi step problems for area and perimeter Know the difference between the two types of special right triangles and their relationships Investigate to find the ratios of SRT's Have previous knowledge of how to solve Pythagorean Theorem problems <u>Misconception:</u>

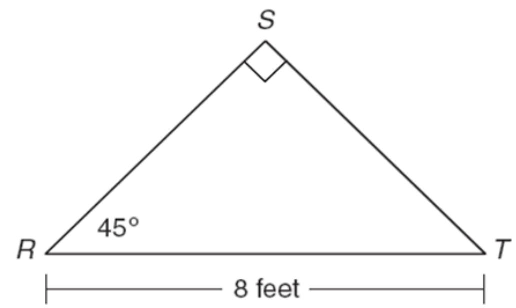
Students should know that if given two congruent sides with included 90 degree angle, then it is a 45-45-90.
Simplifying radicals & rationalize denominator

Examples How to Solve Missing Sides

Short Side	Long Side	Hypotenuse
5		
	$12\sqrt{3}$	
		20
	24	
		$36\sqrt{3}$
$15\sqrt{3}$		

Side	Side	Hypotenuse
7		
		$24\sqrt{2}$
	$24\sqrt{2}$	
		12

<u>Example Problems</u>
The perimeter of a square is 36. What is the length of a side of the square? The length of the diagonal?

	7 Find the area of triangle RST.  **A** 16 ft^2 **B** 32 ft^2 **C** 24 ft^2 **D** 40 ft^2

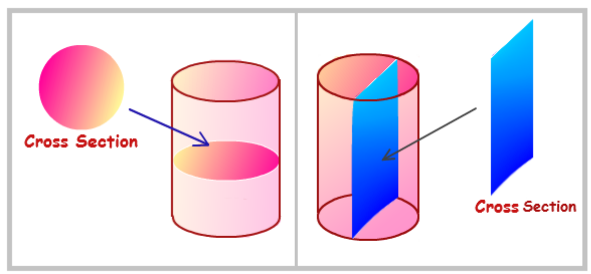
Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
10(A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and	Big Idea - identify shapes made by cross sections and objects generated by rotations Teachers should: Explain what a cross section is Show different ways a figure can be cut in order to create the different shapes Model how three-dimensional objects can be created by two-dimensional

shapes

Students should:
Identify the shape of a cross section
Understand how the cross section changes when cut parallel versus perpendicular to the base.
Understand cutting parallel to the base on a prism or cylinder yields a cross-section that is the same shape as the base.
Understand cutting perpendicular to the base on a prism should always yield a rectangle for the cross-section.
Understand that a cone is created by rotating a triangle, a cylinder is created by rotating a rectangle, and a sphere is created by rotating a circle.

Misconceptions:
Students need to understand that a cross-section is a 2D shape and not a 3D figure.

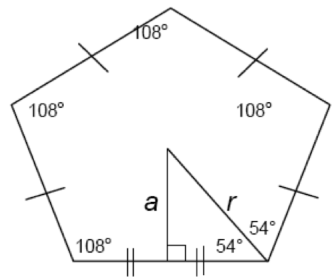
Example:



Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
10(B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.	Big Idea: Changing a dimension affects perimeter, area (or surface area), and volume differently. -Compare and contrast the perimeter, area, surface area, and volume of the new figure with both proportional(all dimensions) AND non-proportional (only some) dimension change. Teachers should: -Use a wide variety of examples that ask students to find the changes in perimeter, area, surface area, and volume. -Use a lot of visuals or manipulatives (i.e. Play-Doh, clay, snap cubes, geoboards, etc.) to talk about how perimeter, area, surface area, and volume change both proportionally and non-proportionally. -Show how students could use this skill to their benefit when problem-solving (i.e. solving an easier related problem). -Use maps, blueprints, or other real-world examples to show square-footage or other ways to think about the standard. -Know that this is typically a low-scoring objective for students and will require more work to help them achieve. -Connect concrete to pictorial to abstract. Students should: -Explore through different types of hands-on activities to best understand this difficult concept. -Use string (or another tool) to cut down a Play-Doh or clay model to understand dimensional change. -Use a ruler (or another measurement tool) to measure dimensions and

	calculate perimeter, area, surface area, and volume of the new figure. -Compare and contrast the perimeter, area, surface area, and volume of the new figure with other examples. -Investigate to find the pattern in proportional dimensional change (x, x^2, x^3) Misconceptions: Students think that a change in the linear dimension affects perimeter, area, surface area, and volume the same. Students can get confused with the non-proportional dimensional change thinking that it is proportional and thus getting the incorrect answer.

Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
11(A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;	Teacher should: Give definition of parts of a regular polygon Show how to create a right triangle using the apothem and a radii Show how to solve for the apothem, missing side length Use trig and/or special right triangles to solve for what is needed Remind students that there are MULTIPLE steps in order to find the area. Make connections in the formula between P and s*n -Multiplying the side length by the number of sides results in the perimeter of a polygon. -Helpful in case students are working backwards for the side length given the area of the polygon



<u>Student should be able to:</u>

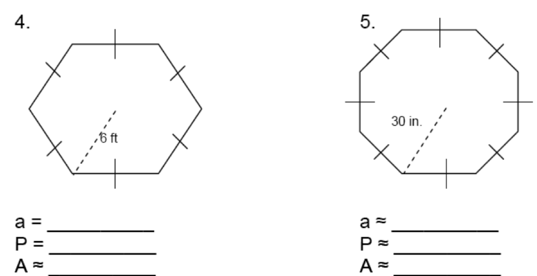
Identify the apothem

Identify all radii and know all radii, sides and angles are congruent.

Use 45:45:90, 30:60:90, and trig. ratios to calculate the triangle created by the radius and apothem.

Use the formula Area=1/2 (Perimeter)(Apothem)

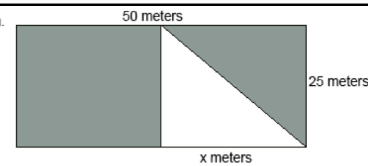
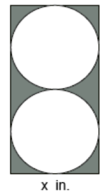
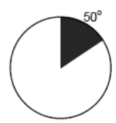
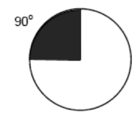
Example Problems w/ Application



<u>Misconception</u>

When students are finding the base of the right triangle, they must double this length to get the side length of the polygon.

Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
11(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;	Big Idea How to find the area of two-dimensional composite figures by separating or joining shapes or parts of shapes Student should be able to: Find area of composite figures (including any combination of triangles, parallelograms, trapezoids, kites, regular polygons) Find area of a sector (slice of circle) Label units of measure Teachers should show: How to break a composite figure into smaller shapes in order to solve. When to subtract the areas versus add the areas Circles: How to set up a proportion to solve for the sector of a circle Examples:

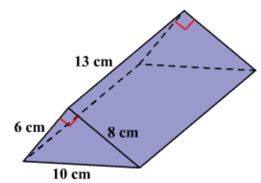
	a.  b.    Misconceptions: Find the area of the shaded region - when you add or when you subtract. Students using the correct units of measure.

Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
11(C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and	<u>Big Idea</u> Find the (lateral and/or total) surface area of three-dimensional figures <u>Students should be able to:</u> Use formulas to solve for lateral and total surface area Prisms, pyramids, cones, cylinders, spheres

Find surface area of composite figures
Choose the correct formula to solve problems
Break apart the composite figures

Teachers should show:
How to find the pieces necessary (length, perimeter base, height, . . .) in order to solve for surface area
How to find given measurement given the surface area
Difference b/t total and lateral surface area
What each variable (letter) stands for in the formula

Examples:
Solve for surface area



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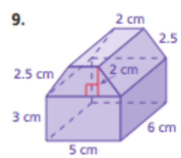

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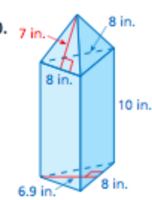
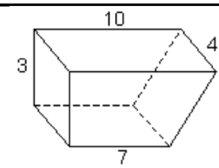
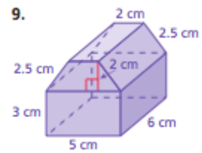
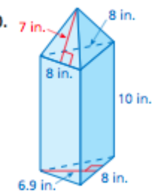


Misconceptions:
P - perimeter of the base
B - area of the base

	Lateral - which sides does it include Where is the base? Are there any parts that you do not need to find the surface area for in the problem?

Two-Dimensional and Three-Dimensional Figures	**Unpacking** What does this standard mean that a student will know and be able to do?
11(D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.	Big Idea Find the volume of three-dimensional figures Students should be able to: Use formulas to solve for volume Prisms, pyramids, cones, cylinders, spheres Find volume of composite figures Choose the correct formula to solve problems Break apart composite figures to find volume Teachers should show: How to use the formulas When to use each formula How to solve for the volume of composite figures How to work backwards for a dimension if the total volume is given What each variable (letter) stands for in the formula Examples: Find the volume for these examples below:

	 9.  10.  <u>Misconceptions:</u> B -Area base What is the base? P - perimeter of the base Units of measurement

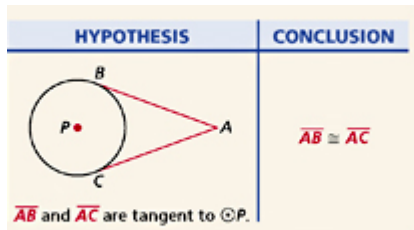
Circles	**Unpacking** What does this standard mean that a student will know and be able to do?
12(A) apply theorems about circles including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;	<u>Big Idea</u> Understanding theorems helps to find relationships between different parts of a circle <u>Teacher should show:</u> -Using definitions, properties, and theorems, Identify and describe relationships w/ inscribed angles, radii and chords. Include central, inscribed, and circumscribed angles.

-Teachers should show how to identify central and inscribed angles and how these angles can help acquire information about arcs.

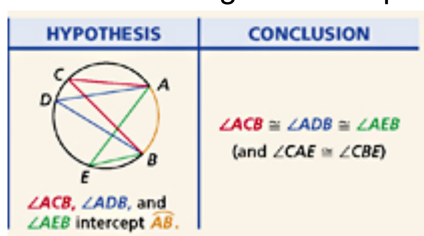
<u>Students should know:</u>

-How to identify and apply theorems:

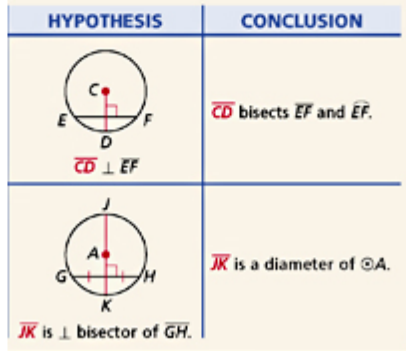
*Two segments tangent to a circle from the same external point are congruent.



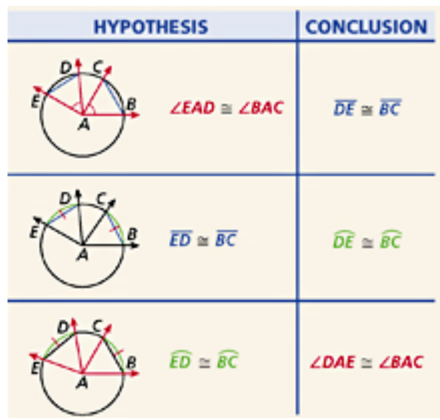
*If inscribed angles intercept the same arc, then the angles are congruent.



*The perpendicular bisector of a chords is the circle’s radius (or diameter).

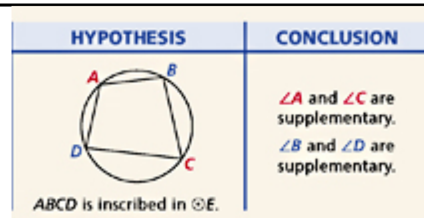


*Congruent central angles, arcs and chords.

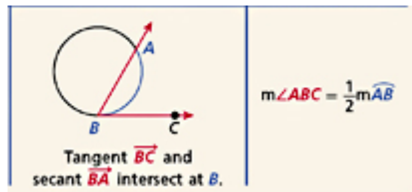


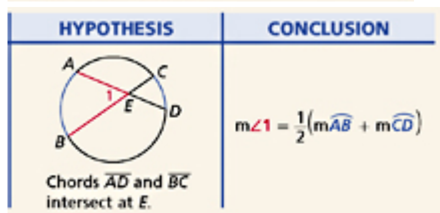
*Inscribed angle theorems:

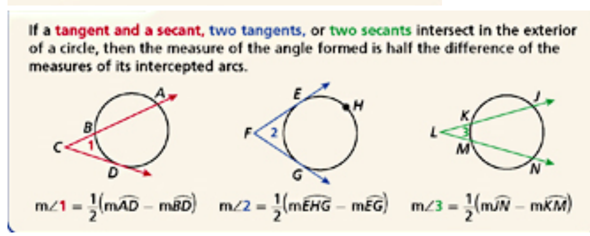
Inscribed angles subtend a semicircle IFF it is a right angle.

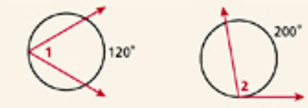
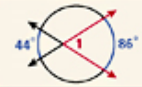
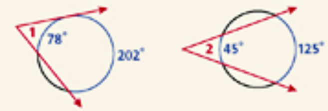


*Theorems with tangents, chords and secants:



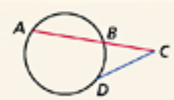




VERTEX OF THE ANGLE	MEASURE OF ANGLE	DIAGRAMS
On a circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$ $m\angle 2 = 100°$
Inside a circle	Half the sum of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(44° + 86°) = 65°$
Outside a circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°) = 62°$ $m\angle 2 = \frac{1}{2}(125° - 45°) = 40°$

HYPOTHESIS	CONCLUSION
 Chords $\overline{AB}$ and $\overline{CD}$ intersect at E.	$AE \cdot EB = CE \cdot ED$

HYPOTHESIS	CONCLUSION
 Secants $\overline{AE}$ and $\overline{CE}$ intersect at E.	$AE \cdot BE = CE \cdot DE$

HYPOTHESIS	CONCLUSION
 Secant $\overline{AC}$ and tangent $\overline{DC}$ intersect at C.	$AC \cdot BC = DC^2$

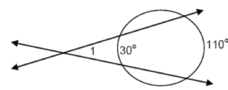
<u>Misconceptions:</u>

Different theorem definitions

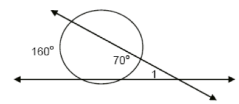
Secant vs. tangent line

Central Angles vs. Inscribed Angles and their relationship with their intercepted arcs

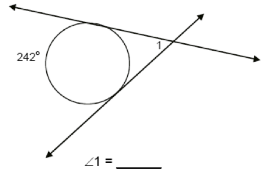
<u>Examples:</u>



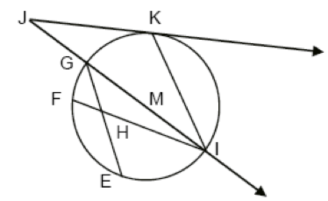
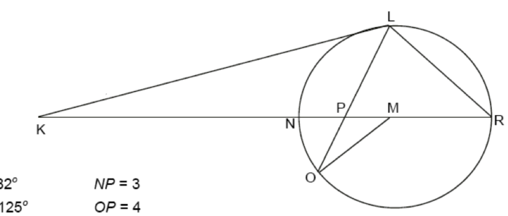
$\angle 1$ = _____



$\angle 1$ = _____

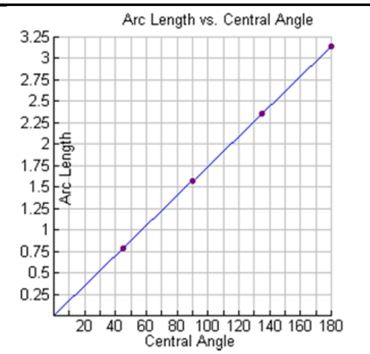


$\angle 1$ = _____

	2.  Given Circle M $m\widehat{GK} = 50^\circ$ $m\widehat{KI} = 4x - 40$ $m\widehat{IE} = x + 45$ $m\widehat{EF} = x + 25$ $m\widehat{FG} = x$ Find a. $m\angle J$ b. $m\angle KIJ$ c. $m\angle JKI$ d. $m\angle IHE$ e. $m\angle GIF$ f. $m\angle FHE$  $\widehat{LN} = 82^\circ$ $NP = 3$ $\widehat{OR} = 125^\circ$ $OP = 4$ $\angle K = 28^\circ$ $PL = 12$ $KN = 30$

Circles	**Unpacking** What does this standard mean that a student will know and be able to do?

12(B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;	<u>Big Idea</u> Arc length is a piece of the circumference and the degree of rotation is a piece of the total of 360 degrees. <u>Teachers should show:</u> Part to whole relationships Arc length is portion of the circle's circumference & can be expressed in terms of π Teachers will need to explicitly show how the relationship is proportional. Example: Use examples to create a linear function that represents relationship. **Degrees Rotation = Central Angle (same thing) $\frac{Arc\ Length}{2\pi r} = \frac{Degrees\ of\ Rotation}{360^\circ}$ <u>Student should know:</u> How to use a proportion to find the arc length given a central angle Find the central angle given the arc length Find the circumference given arc length & central angle Similarity between all circles <u>Examples:</u> Linear relationship to show proportionality



Application:

1. Suppose a professional baseball player swings a bat in an arc that has a radius of 48 inches and sweeps through 255° of rotation. Assuming the ar c is circular, what is the distance the tip of the bat travels to the nearest inch? How many feet is this rounded to the nearest foot?
2. How far does the tip of a 14 centimeter long minute hand on a clock move in 10 minutes?
3. An electric winch is used to pull a boat out of the water onto a trailer. The winch winds the cable around a circular drum of diameter 5 inches. Approximately how many times will the winch have to rotate in order to roll in 5 feet of cable?

Misconceptions:

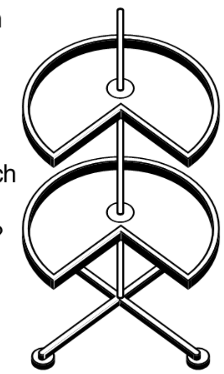
How to manipulate pi

Simplifying proportional

Solving minor arc vs. major arc length

Students can struggle with setting up proportions

Circles	**Unpacking** What does this standard mean that a student will know and be able to do?
12(C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;	Big Idea: Area of a section is a piece of the area of the circle and the degree of rotation is a piece of the total of 360 degrees. Teachers should show: Emphasize the similarity of all circles Note that by similarity of sectors with same central angle, arc lengths are proportional to radius - Basis for introducing radian as unit of measure Not intended that it be applied to development of circular trig in this course. Area of a circle is proportional to the circle's area and can be expressed in terms of *p* $$\frac{Area\ Swept}{\pi r^2} = \frac{Degrees\ of\ Rotation}{360^\circ}$$ Students should know: How to solve for the area of a sector How to solve for the area of a circle Apply proportional relationships to solve for area Similarity between all circles Misconceptions: Setting up proportion Minor sector vs. major sector Simplifying proportion to solve

	<u>Examples:</u> 1. Draw a diagram of a circle with radius 8 units and a sector formed by a central angle of 30. Find the area of the sector. 2. A sprinkler head sprays water a distance of up to 20 feet from the head. Find the area of the sector covered by the spray of the sprinkler head to the nearest square foot. The circular shelves in diagram are each 28 inches in diameter. The “cut-out” portion of each shelf is 90°. Approximately how much shelf paper is needed to cover both shelves? **A** 154 in^2 **B** 308 in^2 **C** 462 in^2 **D** 924 in^2 

Circles	**Unpacking** What does this standard mean that a student will know and be able to do?
12(D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and	<u>Big Idea:</u> Students will be using radians instead of degrees to describe an angle <u>Teachers should show:</u> New to Geometry

Show how to write the measure of an angle in terms of radians
Unit circle as an example

Background:
For this measurement, consider the unit circle (a circle of radius 1) whose center is the vertex of the angle in question.
It is easy to convert between degree measurement and radian measurement. The circumference of the entire circle is 2π , so it follows that 360° equals 2π radians. Hence, 1° equals $\pi/180$ radians, and 1 radian equals $180/\pi$ degrees.

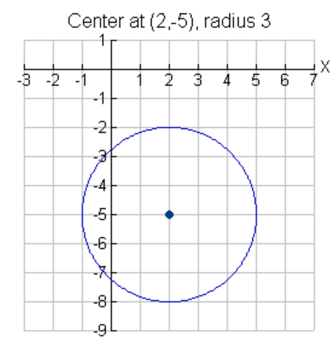
Most calculators can be set to use angles measured with either degrees or radians.
Alternate definition of radians is sometimes given as a ratio. The radian measure of the angle is the ratio of the length of the intercepted arc to the radius of the circle.

For instance, if the length of the arc is 3 and the radius of the circle is 2, then the radian measure is 1.5.

The reason that this definition works is that the length of the intercepted arc is proportional to the radius of the circle.This alternate definition is more useful, however, since you can use it to relate lengths of arcs to angles.

Students should know:
Write the radian measure of an angle
Understanding the ratio of the length of an arc intercepted by a central angle and the radius of the circle

	Radian measure of an angle is the length of the arc intercepted on a circle of radius 1 by an angle in standard position on a coordinate plane. Or equivalently, the radian measure of a central angle in standard position on a coordinate plane is the ratio of the intercepted arc length to the radius of the circle. <u>Misconceptions:</u> What a radian is How radians are different from degrees <u>Examples:</u> 1. Given the length of the arc *l* and the radius *r,* to find the angle at the center. (a). l = .16296, r = 12.587. (b). l = 1.3672, r = 1.2978. What is 70º in radians? To change this degree measurement to radians, we multiply as follows: $70° \bullet \frac{\pi}{180} = \frac{7\pi}{18} \approx 1.222 \text{ radians}$ What is 2 radians in degrees? To change this radian measurement to degrees, we multiply: $2 \bullet \frac{180}{\pi} = \frac{360}{\pi} \approx 114.59°$

Circles	**Unpacking** What does this standard mean that a student will know and be able to do?
12(E) show that the equation of a circle with center at the origin and radius *r* is *x2* + *y2* = *r2* and determine the equation for the graph of a circle with radius *r* and center (*h*, *k*), $$(x - h)^2 + (y - k)^2 = r^2$$	<u>Big Idea</u> Write the equation of a circle. <u>Teachers should show:</u> New to Geometry Expose the formula to students by explaining how r, h, and k can change the shape of the circle Graph the circle Equation: $(x-h)^2+(y-k)^2=r^2$  $(x-2)^2+(y+5)^2=9$ <u>Students should know:</u> How to write the equation for a circle How to find the area of the circle given the equation Identify parts of the equation

	<u>Misconceptions:</u> Students should know that circles can have a center NOT at (0, 0) Students can make a mistake when substituting in a negative value for h or k since the equation has $x - h$ and $y - k$.

Probability	**Unpacking** What does this standard mean that a student will know and be able to do?
13(A) develop strategies to use permutations and combinations to solve contextual problems;	<u>Big Idea:</u> *The difference between permutations and combinations *Permutations are combinations where order is important Key vocabulary: What is a <u>permutation</u>? - way of selecting several things out of a larger group, where order DOES matter **ORDER DOES MATTER - if you change the order it creates a different permutation Examples: - How many 9-player batting orders could you create in a gym class with 32 students? - The number of unique 4-digit PIN numbers you can create to unlock your smart phone. (1234 is unique from 2341, and 3214 etc.) What is a <u>combination</u>? - way of selecting several things out of a larger group, where order does

	not matter. **ORDER DOES NOT MATTER- rearranging the placement of the items does not create a new combination Examples: - How many different pizzas can you make if you are building a 3-topping pizza from 20 topping choices? - How many 9-player teams can you create out of a gym class of 32 students? Teachers should show:/know **Permutations** Are a list All possible ways to rearrange the group Each arrangement is unique **Combinations** Are a group Order does NOT matter Even if you mix up the order, still the same **How to use a permutation to solve ** How to use a combination to solve Student should know: When it is appropriate to use a permutation versus a combination.

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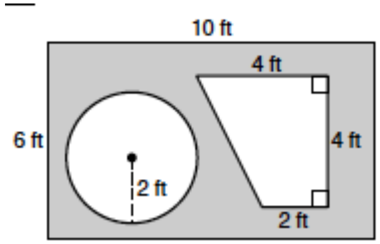
<table>
<tr><th>Probability</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>13(B) determine probabilities based on area to solve contextual problems;</td><td>Big Idea: Relationship between area and geometric probability

Teacher should show:
Finding area of regions in considerations
How to set up a ratio (part out of whole)
Probability can be represented in three ways (fraction, percent, decimal)

Students should know:
How to find area of various polygons and circles
Understand “part out of whole” idea behind probability

Example: Find the probability that a point chosen at random will land in the shaded region.


Connection to context: Connect geometric probability to hitting an archery target.</td></tr>
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Probability	Unpacking What does this standard mean that a student will know and be able to do?
13(C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;	Big Idea: How two individual probabilities are combined to find the probability of one event. Teachers Should: Discuss/show real-life examples where replacement occurs. Discuss/show real-life examples where replacement does not occur. Present the students with different situations in which they need to decide if probability with replacement is appropriate. Students Should: - Be able to decipher between independent and dependent events. -Need to understand the total number of elements to choose from is reduced if an element is not replaced. Common Misconception: “And” means multiply. For example, finding the probability of rolling one die and getting a 4 and rolling another die and getting a 5. Students need to calculate the probability of each event and multiply their probabilities together. Examples: A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die. A card is chosen at random from a deck of 52 cards. It is not replaced and a

	second card is chosen. What is the probability of choosing a jack and then an eight?

Probability	**Unpacking** What does this standard mean that a student will know and be able to do?
13(D) apply conditional probability in contextual problems; and	Big Idea: Unlike independent probabilities, the outcome does rely on the outcome of the previous event. Teachers should: Find the probability of the event given that another event has already occurred Students: Students need to understand the probability of a conditional event assumes that the prior event has already happened Example: The probability that Sue will go to Mexico in the winter and to France in the summer is 0.4. The probability that she will go to Mexico in the winter is 0.6. Find the probability that she will go to France this summer, given that she just returned from her winter vacation in Mexico. A box contains three blue marbles, five red marbles, and four white marbles. If one marble is drawn at random, find: a) P(blue|not white) b) P(not red|not white)

Probability	**Unpacking** What does this standard mean that a student will know and be able to do?
13(E) apply independence in contextual problems.	Independent events in terms of conditional probability -Conditional probability – for two events, A and B, the probability of B given that A has already occurred; written as P(B|A); read as "the probability of B, given A" - Independent probability – if P(B|A) = P(B), then A and B are said to be independent ● Distinguish between events that are independent and dependent using contextual problems involving conditional probability. <u>Example 1</u> A game requires rolling a six-sided die and spinning a spinner. The spinner has four equal sections (red, blue, green and yellow). Here, the probability of rolling a 6 on the die is ⅙ (or, P(6) =1/6 . Also, the probability of the spinner landing on blue is ¼ (or, P(blue) = ¼). The conditional probability P(blue | 6) means to find the probability of spinning the color blue, given that a 6 has already been rolled on the die. Here, P(blue | 6) is the same as P(blue), which is 1/4. In other words, rolling a 6 on the die and spinning blue on the spinner are independent events, because the outcome of one does not affect or influence the outcome of the other. <u>Example 2</u> A box of different colored crayons has one red, one blue, one yellow and one green crayon. From the box, a child randomly selects a crayon, then (without replacement) selects another. On the first selection, the probability

	of each color is $\frac{1}{4}$. Consider, however, P(red | blue), which refers to the conditional probability of pulling a red crayon, given that the blue one has already been selected. If the blue crayon is no longer in the box, then the probability of selecting any of the remaining crayons is now $\frac{1}{3}$. Or, P(red | blue) = $\frac{1}{3}$. Since P(red) is not equal to P(red | blue), these are dependent events. In other words, the first choice affects the possible outcomes of the second event. • Compute compound probabilities based on conditional events, using formulas such as P(A and B) = P(A) P(B|A). <u>Example 1</u> A jar contains 3 red, 2 blue, and 5 green gumballs. A person is to randomly select a gumball, then (without replacement) select another.